MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Professor R. A. Sampson, M.A., D.Sc., F.R.S., PRESIDENT, in the Chair.

Lieut.-Colonel W. A. Tilney, 17th Lancers, Sutton Bonington, Loughborough, Leicestershire,

was balloted for and duly elected a Fellow of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Mary A. Blagg, Greenhill, Cheadle, Staffordshire (proposed by H. H. Turner);

Ella K. Church, Moormead Shaw, Ashtead, Surrey (proposed by T. E. R. Phillips);

A. Grace Cook, The Rookery, Stowmarket, Suffolk (proposed by W. F. Denning);

Frederick Charles Leonard, President of the Society for Practical Astronomy, 1338 Madison Park, Chicago, U.S.A. (proposed by F. R. Moulton);

Percy Austin Openshaw, B.A., Headmaster, Belmont, Blackheath, Belmont, Lee Terrace, Blackheath, S.E. (proposed by F. W. Dyson);

Ludwik Silberstein, Ph.D., Lecturer in Natural Philosophy in the University of Rome, 4 Anson Road, Cricklewood, London, N.W. (proposed by Alfred W. Porter);

Julius F. Stone, President of the Seagrave Company, Manufacturing Engineer, Columbus, Ohio, U.S.A. (proposed by H. C. Lord);

Irene Elizabeth Toy Warner, Ardagh, Horfield Common, Bristol (proposed by W. F. Denning); and
Mrs. Fiametta Wilson, Ansol Road, Bexley Heath, Kent (proposed by J. R. Leeson).

One hundred and ten presents were announced as having been received since the last meeting, including, amongst others:—

Portraits of Sir William and Lady Huggins (oil paintings), and MS. Observing books of the Rev. W. R. Dawes, presented by the Executrix of Lady Huggins.

G. E. Hale, Ten Years' Work in a Mountain Observatory, presented by the author; Cape Astrographic Zones, vols. 2, 3 (Zones $-42°$ and $-43°$), presented by the Royal Observatory; Astrographic Chart: Eighty-three charts, from the Observatories of Algiers, Bordeaux, Paris, and Toulouse, presented by the French Minister of Public Instruction.

Regimento do Estrolabio e do Quadrante, and P. Nunez, Tratado da Sphera (reproductions in facsimile), presented by the Portuguese Government.

Note by the Secretaries on Father Rigge's Paper on the Solar Eclipse of 1916 *December* 24 (vol. lxxv. p. 687).

Father Rigge mentions that no map of this eclipse is given in the *American Ephemeris*. There is also no map in the *Nautical Almanac*. Professor Andoyer has kindly pointed out that a map of the eclipse was given in the *Connaissance des Temps* for 1916.

A Proposal for the Comparison of the Magnitude Scales of the Astrographic Catalogue. Eighth Note: The Cape Magnitudes for $-42°$. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The seventh note is in *Monthly Notices*, lxxv. p. 601, and the sixth in lxxv. p. 465, where previous references are given. The Cape Magnitudes for $-41°$ were studied in the fourth note (lxxv. p. 57), and the main interest of the present note is the confirmation of the result previously obtained for the limits of the "obscured patch in the sky" as sketched in the sixth note. For this purpose it is not necessary to make the counts in full detail: it will suffice to make them for two points of the scale. For one of these we may take "All" stars on the plates: the counts are then printed for each plate in the Cape volume, and we have only to add them up for each hour. For the other point we take the transition from measures to estimates: the latter are prefixed in the printed columns by a negative sign, and the number of

negative signs can be counted with comparative ease. These counts were kindly made by Mrs. W. F. R. Weldon.

2. In the following table the counts for each hour are given. The intervals between plate centres being 10 minutes, there are just six plates in each hour: centres at 0^m, 10^m, 20^m, 30^m, 40^m, 50^m: mean 25^m. But the difference between this and 30^m has been neglected in forming the galactic latitude (from *Gron. Pub.*, No. 18) for the middle of each hour. The earlier columns are sufficiently explained by their headings.

TABLE I.

Counts of Cape Zone – 42°.

R.A.	M = Measured Stars.	A = All Stars.	Log $\frac{A}{M}$.	Diff. from Mean.	Zone $-41°$.	Gal. Lat. for $-42°$.	Zone $-41°$ IV. + III. − II.
0^h	390	805	0·31	− ·09	− ·09	− 75°	− 29
1	551	935	·23	− ·17	− ·03	− 72	− 18
2	446	1078	·38	− ·02	+ ·02	− 63	− 22
3	579	1200	·32	− ·08	− ·01	− 53	− 46
4	862	1711	·29	− ·11	− ·07	− 42	− 9
5	1345	2747	·31	− ·09	− ·08	− 31	+ 4
6	1540	3656	·37	− ·03	− ·05	− 20	− 9
7	2071	4811	·36	− ·04	− ·07	− 10	− 3
8	2758	5935	·33	− ·07	− ·13	0	− 42
9	1810	3984	·34	− ·06	+ ·02	+ 7	− 9
10	1854	4479	·38	− ·02	+ ·02	+ 14	+ 27
11	851	2812	·52	+ ·12	+ ·04	+ 19	+ 5
12	617	1899	·47	+ ·07	− ·02	+ 21	+ 4
13	960	2767	·46	+ ·06	+ ·08	+ 20	+ 16
14	964	3156	·52	+ ·12	+ ·13	+ 16	+ 36
15	1378	4818	·54	+ ·14	+ ·15	+ 10	+ 35
16	1721	5094	·47	+ ·07	+ ·12	+ 3	+ 12
17	2814	9880	·54	+ ·14	+ ·09	− 6	+ 48
18	3612	12178	·53	+ ·13	− ·05	− 16	+ 18
19	1296	4301	·52	+ ·12	− ·09	− 26	+ 22
20	835	2313	·44	+ ·04	− ·02	− 37	+ 2
21	636	1768	·45	+ ·05	+ ·02	− 49	+ 14
22	557	1046	·27	− ·13	− ·01	− 60	− 11
23	376	862	·36	− ·04	− ·09	− 70	− 44
Sums and Means	30853	84230	·40				

3. It will be seen from comparison of the corresponding results for $-41°$ given in the sixth column that the changes from negative to positive and back again occur at approximately the same places in both columns. The results for zone $-41°$ do not appear explicitly in the fourth note (*M.N.*, lxxv. p. 63). They are the

differences between the columns headed III. and IV. In that table a rather more elaborate determination of the obscuration is made by combining several columns: it is headed IV. + III. − II., and is repeated for convenience in the eighth column of Table I., so as to show how the heavy lines for zone − 41° were determined. It is not possible to make a corresponding combination for zone − 42° without more detailed counts, which at present have not been made; but the simple counts already available seem to show the breaks for − 42° clearly, and suggest that while that from + to − is in much the same place as for − 41°, viz. between 21^h and 22^h, that from − to + is distinctly later. On reference to the diagram of suggested "Obscured Patches" (*M.N.*, lxxv. p. 480), it will be seen that this accords very well, by extrapolation, with the Perth-Cape results. The boundary line is travelling rapidly to greater R.A.'s as we go south.

4. It may be mentioned here that Mr. R. J. Pocock has already obtained evidence of the sharp boundary near 6^h in − 17°, midway between Perth and Algiers. At the Nizamiah Observatory, Hyderabad, they have now a continuous series of plates, measured and reduced, from 3^h to 13^h in zone − 17°, and the counts on them give results as below, denoting by N'_d the total number of stars with measured diameter greater than d.

TABLE II.

Hyderabad, Zone − 17°. *Unit* 0·01.

R.A.	Log N_{50}.	Log N_{30}.	Log N_{20}.	Log N_{12}.	Log N_8.	Gal. Lat.	
h						°	
3	− 1	0	+ 5	+ 2	− 8	− 50	Obscured.
4	+ 9	0	+ 2	− 1	− 1	− 36	
5	+ 1	0	− 13	− 25	− 36	− 23	
6	+ 4	0	− 4	− 11	− 17	− 10	
7	− 16	0	+ 10	+ 13	+ 16	+ 3	Unobscured.
8	− 10	0	− 2	− 1	+ 3	+ 15	
9	− 1	0	+ 1	0	+ 3	+ 26	
10	− 8	0	− 6	− 2	+ 1	+ 36	
11	+ 6	0	+ 11	+ 16	+ 22	+ 42	
12	− 6	0	− 3	+ 7	+ 5	+ 45	
13	+ 22	0	− 2	+ 2	+ 12	+ 45	
C. and M. scale	9·66	10·73	11·60	12·34	12·90		

He proceeds to deduce the galactic condensation, paying attention to the existence of the "obscured patch," and finds confirmation of the suggested increase as we go southwards. It is of interest to mention this additional evidence; but Mr. Pocock will no doubt prefer to reserve the full details until the whole zone is finished.

5. Returning now to the results of Table I., column 5, we may arrange them in the order of galactic latitude, keeping the obscured and unobscured portions separate. Alongside the results for − 42° we will place those for − 41° derived from column 6. The galactic latitudes are not quite the same for this column, but we will neglect the difference. We must, however, omit the results

for 10^h, which are assigned to different portions in the two zones. It will suffice to give the means of groups as follows:—

TABLE III.

Deduction of Galactic Condensation.

"Obscured."				"Unobscured."			
Mean Gal. Lat.	No. in Group.	Mean Log Ratio. −42°.	Mean Log Ratio. −41°.	Mean Gal. Lat.	No. in Group.	Mean Log Ratio. −42°.	Mean Log Ratio. −41°.
68°	5	−·090	−·040	37°	3	+·070	−·030
42°	3	−·093	−·053	20°	3	+·083	+·033
9°	4	−·050	−·058	10°	5	+·120	+·088

There is thus no evidence of sensible change of the log ratio in high galactic latitudes. Confining attention to latitudes less than 50°, and noting that the magnitude difference represented in columns 5 and 6 of Table I. is 1·23, we get for the condensation per magnitude, for 1° of galactic latitude,

For "obscured" (42° and 9°) − ·043/33 ÷ 1·23 = − ·00106 for − 42°, + ·00012 for − 41°
For "unobscured" (37° and 10°) − ·050/27 ÷ 1·23 = − ·00150 for − 42°, − ·00355 for − 41°.

The results for − 41° are not quite the same as those given in *M.N.*, lxxv. p. 608, viz. − ·00046 and − ·00390, since these were found from the more elaborate combination shown in column 8 of Table I.; but the very large coefficient for "unobscured" duly appears. The better accordance of the two results for − 42° suggests that the discrepancies in − 41° are largely accidental.

6. The general result is that the value of $\log N_{m+1} - \log N_m$, where N_m is the number of stars brighter than magnitude m, is sensibly constant near the galactic poles and increases near the Galaxy at about ·0012 per degree of galactic latitude. We can make a smooth curve to represent this increase by taking such a function for the rate of increase as $C \cos^4\theta$, where θ is the galactic latitude. By way of illustration, let us assume that the condensation is somewhat as follows:—

Gal. lat.	90° to 70°	60°	50°	40°	30°	20°	10°	0°
Condensation per mag.	·000	·001	·004	·009	·018	·029	·043	·058
For 1·23 mag.	·000	·001	·005	·011	·022	·037	·053	·071

7. Let us apply these corrections as in Table IV. The second column repeats column 5 of Table I. The next column gives the correction for galactic latitude which is applied in the fourth column. Now one of the points on which emphasis has been laid is that the transitions from the "obscured" regions to the "unobscured" are abrupt; and it seems worth while to bring out this point as clearly as possible. The numbers of column 5, it is submitted, are like the series

− 1 − 1 − 1 − 1 − 1 + 1 + 1 + 1 + 1 + 1 + 1 − 1 − 1 − 1 − 1

with two definite jumps, and uniformity between the jumps. The uniformity and the jumps may both be masked by accidental

errors, but they are real; the series is not for instance like a sine-curve. To bring out this character we have to reduce the accidental errors, which means taking several terms together. Suppose now in the above type-series we subtract the sum of the three terms preceding any individual from the sum of the three terms following it. Then so long as we have uniformity, whether of −1 or of +1, we get zero difference. But near a jump we get as successive terms

. . . 0 0 +2+4+6+6+4+2 0 0 . . .

and near the other jump we get of course negative signs for the same numbers. This operation is performed on the fourth column in the fifth. The jumps are brought out very clearly, and the magnitude of each is readily deduced from the largest numbers, which should be 3 times the jump. We may thus put the jump at 16, which means ·016. Treating the corresponding numbers for −41° in the same way (columns 7 and 8), we do not get at all a clear answer, as indeed is seen on inspection of the results as they stood originally. But if we use the combination shown in the last column of Table I. (which corresponds to a magnitude difference of about 4·8 and has been corrected accordingly), the jumps come out clearly and correspond in position closely to those of zone −42°.

TABLE IV.

Hour.	Col. 5 of Table I.	Corrn. for Gal. Lat.	Sum.	Follg. 3 − Prec. 3.	Zone −41° Corr.	Follg. 3 − Prec. 3.	Zone −41° IV.+III.−II. Corr.	Follg. 3 − Prec. 3.
0	− 9	0	− 9	−14	− 9	+ 7	−29	− 41
1	−17	0	−17	+ 4	− 3	+12	−18	+ 3
2	− 2	0	− 2	− 1	+ 2	+ 2	−22	+ 28
3	− 8	0	− 8	− 2	− 1	−17	−46	+ 27
4	−11	−1	−12	0	− 8	−29	−13	+ 34
5	− 9	−2	−11	− 8	−10	- 34	− 4	− 37
6	− 3	−4	− 7	− 4	− 9	−17	−25	− 63
7	− 4	−5	− 9	− 3	−12	0	−23	− 54
8	− 7	−7	−14	+16	−20	+24	−70	+ 15
9	− 6	−6	−12	+34	− 4	+32	−33	+102
10	− 2	−5	− 7	+48	− 3	+34	+ 7	+103
11	+12	−4	+ 8	+46	0	+34	−11	+104
12	+ 7	−4	+ 3	+30	− 6	+30	−12	+ 72
13	+ 6	−4	+ 2	+13	+ 4	+33	0	+ 35
14	+12	−4	+ 8	+ 4	+ 9	+16	+20	+ 46
15	+14	−5	+ 9	+ 4	+10	− 8	+15	+ 2
16	+ 7	−7	0	+ 7	+ 5	−41	−16	+ 1
17	+14	−6	+ 8	+ 4	+ 3	−48	+24	− 9
18	+13	−4	+ 9	− 1	− 9	−32	+ 2	− 5
19	+12	−3	+ 9	−23	−12	− 2	+10	− 13
20	+ 4	−1	+ 3	−39	− 3	+ 9	− 2	− 81
21	+ 5	−1	+ 4	−47	+ 1	+ 5	+10	− 94
22	−13	0	−13	−46	− 1	− 7	−11	−109
23	− 4	0	− 4	−22	− 9	− 7	−44	− 66

Summary.

The Cape count for $-42°$ confirms the conclusions previously deduced from $-41°$. The "obscured" patch in these declinations extends from 22^h through 0^h to 10^h; and the galactic condensation found from these zones is such that near the Galaxy log N increases at a rate rising to ·0015 per magnitude for one degree of galactic latitude.

Incidentally it is remarked that Mr. Pocock has already obtained evidence of one limit of the "obscured" patch in zone $-17°$ near 6^h–7^h, from counts on Hyderabad plates 3^h–13^h.

Note added November 14.—The above figures were put together in September, but the paper was just too late for the Supplementary Number. Since then other counts have been tabulated for Paris $+22°$ and Toulouse $+9°$, giving results which fall in with those shown in the diagram of the sixth note (*M.N.*, lxxv. p. 480). It will probably be convenient to collect as many zones as possible before repeating this diagram, but a simplified conception of it may be mentioned here. The obscured patches form a spiral round the celestial sphere, the median line of which follows approximately the curve

$$\alpha + 4\delta = \text{constant } (265° \text{ approx.})$$

where α is the R.A., and δ the north declination. Thus in the present paper the obscured patch extends from $22^h{\cdot}0$ to $11^h{\cdot}0$, the middle of which is $\alpha = 4^h{\cdot}5 = 68°$: and $\delta = -42°$. Thus

$$\alpha + 4\delta = 428° - 168° = 260°.$$

For the Perth magnitudes the obscured patch is confined to the portion $23^h{\cdot}0$ to $4^h{\cdot}0$, the middle of which is $\alpha = 1^h{\cdot}5 = 23°$, and $\delta = -32°$; so that

$$\alpha + 4\delta = 383° - 128 = 255°.$$

For Bordeaux, obscured patch is $7^h{\cdot}0$ to $20^h{\cdot}0$, $\alpha = 203°$, $\delta = +17°$,

$$\alpha + 4\delta = 203° + 68° = 271°.$$

For the Vatican (*M.N.*, lxxv. p. 604) obscured patch is $20^h{\cdot}0$ to $6^h{\cdot}0$, $\delta = +62°$

$$\alpha + 4\delta = 30° + 248° = 278°.$$

These results are taken direct from figures already published. The cases of Oxford and Algiers raise the question of treatment when the obscured patch has gaps, and discussion of this point is reserved. A slight change in the coefficient of δ would clearly improve the accordance of the constants, but the inclination of the spiral to our equator must be close to 15°.

Some Questions relating to the Shape of the Earth, suggested by Mr. Harold Jeffreys' Paper, "Certain Hypotheses as to the Structure of the Earth and the Moon" ("Memoirs of Royal Astronomical Society," vol. lx. part vi.). By Arthur R. Hinks, M.A., F.R.S.

I beg leave to offer to the Society some remarks upon the second chapter of Mr. Jeffreys' paper, in which he discusses the departure of the earth's surface from the hydrostatic form. It appears to me that his treatment of the subject must present some difficulty to the geodesist or to the geographer: a difficulty which may perhaps be resolved most easily if the author will be so good as to explain rather more fully the physical conception underlying his mathematical treatment of the subject.

1. The chapter opens thus: "The deviation of the land's surface from the hydrostatic form can be measured directly, since the ocean's surface has the hydrostatic form." If by hydrostatic form is meant merely some equi-potential surface, this is necessarily true; but it cannot be without interest to inquire what figure is intended. The hydrostatic form is not a regular spheroid. How much it may depart from the spheroid depends principally upon the adequacy of deficiencies of the density below the visible masses, and excesses of density in the ocean floor, to balance the disturbances of figure due to the visible excesses and defects. It depends, in short, upon the condition known as isostasy. It is generally believed that the isostatic compensation is so far complete that the figure of the ocean's surface does not differ from the spheroid by anything like as much as the hundreds or even thousands of feet which were calculated before the principle of compensation was established. But at the same time the evidence is probably too scanty to enable anyone to affirm that the ocean's surface, or the geoid, and the spheroid which fits it most nearly do not depart from one another in places by perhaps a hundred feet or more.

There is at present a tendency to assume that isostasy is more complete than the data will really warrant. The most elaborate investigation of the subject—that of Hayford—is limited to the comparatively small area of the United States, and it does not seem to the writer that Hayford has sufficiently established his principal thesis: that there is a definite depth at which isostatic compensation is complete. Recent investigations of the Survey of India have shown, moreover, that whereas there is without doubt a considerable compensation under the Indian Peninsula, it is not anywhere complete, but is much more complete under the older rocks than under the newer. It would seem therefore that, while the doctrine of a considerable and even large degree of compensation is fairly established in such cases as have been investigated, it is much too soon to be sure that the form of the

ocean's surface is not affected to the extent of a great many feet by the irregular density and configuration of the crust.

Mr. Jeffreys proceeds to take the elevation of the crust above sea-level and the depth of the ocean floor below it at about three hundred points, and analyses these divergences from the hydrostatic form after multiplying all depths below sea-level by the factor $\frac{7}{11}$ths "to allow for the weight of the water itself, which is capable of producing a considerable reduction of strain as compared with what the strain would be were the ocean absent." This procedure seems to require some explanation. If the author has accepted, without mentioning it, that isostatic compensation is at any rate moderately complete, then is it not the case that the effect of the water is already involved in the assumption, and that to introduce a correction for it explicitly is to take account of it a second time? If, on the other hand, it is correct to take account of the water in this way, does it not lead to a presumption that Mr. Jeffreys does not admit the condition of isostasy, but must be prepared to allow the full effect of the attraction of uncompensated visible masses and effects?

2. By his analysis of heights and depths at three hundred points Mr. Jeffreys determines "a difference in ellipticity," defined as the coefficient of a second harmonic. The geodesist and the geographer will now inquire what physical conception can be formed of an ellipticity thus defined. I understand that if a solid is not a spheroid its "ellipticity" can be only the ellipticity of that spheroid which most nearly fits it according to a least square criterion, such as the use of the spherical harmonic analysis provides. But it will be remembered that the quantities analysed are divergences not from a spheroid, but from the geoid, and we are not sure that the geoid does not differ essentially from the spheroid. It would seem at first sight that in this case the least square analysis must have been trying to fit the solid surface to a figure which is not a spheroid, and the conception of ellipticity as equivalent to the coefficient of one of the second harmonics becomes more complex; worthy perhaps of a fuller explanation than is afforded by Mr. Jeffreys' severe economy in words.

3. Turn now to a question of geography. In the table on p. 191 Mr. Jeffreys gives the heights and depths for some three hundred different points, without indication of his authorities; and I should wish to remark that the figures are far from representing our present knowledge of geography. For Mr. Jeffreys the explorers of the Arctic and the Antarctic have toiled in vain. He treats as unknown both the depth of the Arctic Sea, though Nansen found depths of 12,000 feet over a large area which probably extends to the North Pole; and as equally unknown the height of the Antarctic plateau, which by the observations of Scott and Shackleton rises to the height of about 10,000 feet over a very large area.

The rest of his points are distributed with geometrical regularity at intervals of 20° of longitude and 10° of latitude over the

earth, without any regard at all to the actual configuration of the land or the ocean. Thus it happens that he ignores the whole Cordillera of the Andes, which by bad luck runs almost due north and south between his chosen meridians of 60° and 80° west of Greenwich. This is an extreme example of the result of sampling the earth at a very restricted number of points; but it is easy to find other examples only slightly less curious. There is perhaps not enough material yet available to make it possible to deduce a mean elevation or depression for each compartment defined by the points of Mr. Jeffreys' selected network. But it would certainly be possible, without serious difficulty, to arrive at a result giving a far more adequate representation of the actual earth than is given by his three hundred samples; and in particular it would be easily possible to obtain a far more accurate representation of the Polar regions than is provided by the long string of queried figures along the parallel of 80° south, as given in the table on p. 191.

It may be noted also that in the table of heights and depths on p. 191 of the memoir we have

Depth	0	14 points
	50 to 11,500	99 ,,
	12,000	19 ,,
	Below 12,000	79 ,,

Giving half the zero points to the sea, and dividing the points given as 12,000 equally on each side of this number, we have $56\frac{1}{2}$ per cent. of the ocean depths less than 12,000 feet and $43\frac{1}{2}$ per cent. greater; whereas according to Murray the percentages should be about 35 and 65 respectively. There is thus a very considerable under-representation of the ocean depths.

4. The result of the harmonic analysis is that the ocean's surface is slightly more elliptic than the lithosphere, by 1 part in 31,300. Granting that this very small quantity may be a spurious result of inaccuracies in his method of combining heights, our author concludes that "the surface of the lithosphere has at present the hydrostatic form to a high degree of accuracy." And the appearance of accuracy is much increased by the statement that the probable error of his determination of the second harmonics is of the order of thirty feet. Mr. Jeffreys does not give the residuals of his solution; but as the resulting coefficient is so minute it would seem that they cannot be very different from the numerical terms with which he started. Consider then for a moment the quantities in the last column of his table, which represent the mean heights for each parallel of latitude after the above-mentioned correction for the weight of the water. The sum of the quantities north of the Equator is $-16{,}139$; south of the Equator it is $-35{,}956$; that is to say, the greater part of the positive residuals would be on one side of the Equator, and of the negative residuals on the other. In an ordinary least square

solution one would view with grave mistrust a result in which the residuals were distributed so unsymmetrically. May one put this mistrust aside when one uses the particular kind of least square solution called spherical harmonic analysis?

This brings us back to our early difficulty: In what sense does Mr. Jeffreys use the word "ellipticity"? It is a geographical commonplace that the distribution of land and water is unsymmetrical with respect to the Equator. There is a great excess of land in the Northern Hemisphere, with a deep sea at the North Pole. There is a great excess of deep water in the Southern Hemisphere with a very high plateau extending, roughly, 10° round the South Pole. The radius of the lithosphere is short by about 1 part in 2000 at the North Pole, and long by about the same amount at the Southern; while in lower latitudes it is more or less systematically long in the Northern Hemisphere and short in the Southern. Neither is the earth at all symmetrical in longitude. There is a great excess of land in what we call the Eastern Hemisphere; a great excess of sea in what we call the Western. We have been invited to think of the earth as a degraded tetrahedron, or as a pear-shaped figure with the stalk at the Southern Pole. The mere fact that these figures have been used to describe the lithosphere might be held sufficient to show that it has no visible appearance of a close fit with the spheroid.

What meaning, then, are we to attach to a determination of ellipticity, with a body so unsymmetrical about the major axis of the ellipse to be determined? It appears that those who speak of the ellipticity of the lithosphere and its close fit with the hydrosphere cannot have realised that the irregular inequalities in the former are of the same order as the regular inequalities in the latter which distinguish it from the sphere and give it an eccentricity. The Polar radius of the hydrosphere is some 70,000 feet shorter than its Equatorial radius. The vertical interval between the highest mountain and the lowest depth of ocean is 60,000 feet —not very much less than the 70,000 feet just mentioned. These are extremes, and a man who had not gone into the geography of the matter might be tempted to say that a few such extreme cases should not be held to affect the general figure of the earth. In reply to such an argument I would call attention to a fact which is frequently overlooked, that the divergences of the lithosphere from sea-level, or from a mean surface parallel to it corresponding as nearly as may be to the mean level of the lithosphere, are not by any means distributed according to the theory of errors.

The mean level of the lithosphere is about 9000 feet below sea-level. Between this and sea-level lies about 22·2 per cent. of the whole area of the lithosphere; while between 9000 and 18,000 below sea-level lies 39·5 per cent.—very nearly twice as much. On the other hand, between sea-level and 6000 feet above lies 25·4 per cent. of the lithosphere, while in the corresponding 6000 feet of vertical interval on the other side of the mean—that is to say, between 18,000 and 24,000 feet below sea-level—there is in-

cluded only 3·1 per cent., or less than one-eighth of the amount on the upper side of the mean.

The areas outside these ranges are comparatively insignificant, and the whole of the values are shown in the following small table, which is extracted from the work of Murray and Hjort, *The Depths of the Ocean*, and from Chamberlin and Salisbury, *Geology*, vol. i.

Above 24,000 feet	0·004	per cent.	of the total area.
From 24,000 to 18,000 feet . .	0·086	,,	,,
From 18,000 to 12,000 ,, . .	0·61	,,	,,
From 12,000 to 6000 ,, . .	1·6	,,	,,
From 6000 feet to sea-level . .	25·4	,,	,,
Sea-level to −6000 feet . . .	14·8	,,	,,
From −6000 to −12,000 feet .	14·8	,,	,,
From −12,000 to −18,000 ,, .	39·5	,,	,,
From −18,000 to −24,000 ,, .	3·13	,,	,,
Below −24,000 feet . . .	0·07	,,	,,

If in the section −6000 to −12,000 we divide the 14·8 into two halves, we find that the total percentages above the dividing line add up to 49·9 and those below to 50·1, so that it is evident that 9000 feet is a close approximation to the mean depth of the lithosphere below sea-level.

Now I would ask whether a distribution like this is a fit subject for investigation by the least square method, without a careful discussion of the physical meaning of the deviations from a normal distribution. There is the double dissymmetry: first of all, in the abnormal distribution of quantities about the mean; and secondly, the abnormal distribution about the equatorial plane of the proposed spheroid. If one requires proof that the lithosphere's resemblance to a spheroid is somewhat slight, here, I think, it is. For my own part I should be as well content with the simpler test that you have only to look at it to see that it is not a spheroid, or anything like one.

5. We come last to a matter which is beyond the scope of the geographer and the geodesist, though it is one in which they must take a general interest, because it concerns the use which is being made of the data which they have been at pains to provide. I am given to understand that Mr. Jeffreys would be likely to say: "I am interested in a certain quantity which I need in my application of Herglotz' equations to the problem I am investigating. This quantity depends upon the coefficient of a second harmonic, and all that I am concerned with is the fact that this second harmonic is small. I am not in the least concerned with the fact that the actual and visible shape of the lithosphere suggests that if it is to be represented by harmonics at all, it will be necessary to take account of harmonics higher than the second. It is sufficient for me that the second harmonics are small. From my

point of view, the surface of the lithosphere has at present the hydrostatic form to a high degree of accuracy." For the reasons which I have indicated above, this conclusion must appear paradoxical, a statistical fiction. It would help to solve our doubts if Mr. Jeffreys would be so good as to explain if his interpretation of the smallness of the second harmonic is unique. Is it not possible that the second harmonic is a vanishing quantity because the lithosphere has *no* close relation to the hydrostatic form?

If one analysed harmonically the divergence of a square from the inscribed circle, would the second harmonic be small? And if so, would it be correct to say that the square and the circle were a very perfect fit? Or again, returning to the actual case, supposing that Mr. Jeffreys had been trying, after the manner of Professor Love, to determine the harmonics—be their order what they may—which best represent the lithosphere, would his present determination of the second harmonic hold good? Or would it be involved in and affected by the determining of the higher harmonics? Or again, if in his analysis he had chosen a different plane of reference, so that the doubtful quantities at the Poles had not been conveniently multiplied by the factor zero, would he still have obtained a similar result? Or, to put this last question in a different way, when you are determining the ellipticity of the mean meridian section of the lithosphere, is it a matter of indifference that you do not know or do not use the actual minor axis?

These questions are no doubt easy to answer, and they merely reflect the ignorance of the questioner. But many geographers and geodesists who read Mr. Jeffreys' paper will be puzzled by the use of the facts which the author has made. They will put these questions to themselves and may not be able to answer them. They will be sincerely grateful for an explanation of the whole matter.

London:
1915 *November* 6.

Note added 1915 *November* 20.—In view of a question asked during the discussion following the reading of the above, it may not be superfluous to recall the fact that geodetic bases are always reduced to mean sea-level, and that all geodetic triangulation is thereby automatically referred to the same level. The resulting "figure of the Earth" is therefore, to a close approximation, the figure of the hydrosphere, and has little or nothing to do with the figure of the lithosphere.—A. R. H.

Viscosity in Relation to the Earth's free Precession.

By Sir Joseph Larmor, D.Sc., F.R.S.

One of the results of Mr. H. Jeffreys' interesting paper on this subject (*Monthly Notices*, June 1915, p. 648) is so surprising that some way out of it seems to be necessary: as, in fact, he fully recognises. He calculates that an internal viscosity in the Earth sufficient to account, by the lag of the solid tides, for the secular acceleration of the Moon's mean motion, would be sufficient to stop down any free precession of the Pole in a few days. Assuming the calculations to be correct, are his physical hypotheses stretched beyond their range of validity?

The law of viscosity that is assumed consists, following G. H. Darwin, in replacing the rigidity n of the material by

$$n\left(1 + 1/\mathrm{t}\frac{d}{dt}\right)^{-1}$$

where t may be called time of relaxation of the substance. This hypothesis probably gives a fair representation of the effects of internal friction for oscillations whose period is very short compared with t. It also embraces, as was doubtless intended, the facts that pitch has fluid properties for disturbances of very slow period, while it can be moulded into a tuning fork for sound vibrations: but ordinary solids do not at all show this fluidity for very slow changes of stress, at any rate for stresses so small, due to a tidal head of a few feet of water, as we are here concerned with.*

On the basis of analysis adapted to the case of a complex n from Thomson and Tait's *Nat. Phil.*, Mr. Jeffreys finds (p. 651) that t must be so small as about a day and a half, in order that the lag of the solid terrestrial tide may produce sufficient drag on the Moon. But the period of the tide is half a day, which is probably much too near to this value of t for the assumed law of viscous elasticity to have any validity.

Moreover, he then applies it to the free precession of the Pole, where the period involved is over 400 days; and this will appear on reflection to be very far beyond its possible range—whether we assume fluidity for very slow stresses, and consequent precise isostasy throughout the Earth's mass, or not. For example, in the case of a maintained vibration of plane shear in a slab of the

* If the elasticity of the core were perfect, the lunar drag on the Earth's rotation must arise from the lag of the oceanic tides alone. It has not yet been proved that this cause is insufficient to account for the lunar acceleration: *cf. Monthly Notices*, Jan. 1915, p. 215, and on the other side Poincaré, *Théorie des Marées*.

But a very slow absorption of energy of tidal strain in the core is certainly not precluded, especially if it is of the type of the elastic fatigue investigated for the case of metals by Kohlrausch, Kelvin, and others.

material, everything would vary as $e^{\omega t}$ where here $2\pi/p = 400$ days: thus the effective value of the rigidity would be $n(1-42\iota)^{-1}$, or $\frac{n}{42^2}+\frac{n}{42}\iota$, where $\iota=\sqrt{-1}$, for a period of 400 days, as contrasted with the full value n for sound vibrations—an entirely improbable extrapolation.

Cambridge:
1915 *Oct.* 16.

The Efficiency of Sun-Spots in Relation to Terrestrial Magnetic Disturbances. By Rev. A. L. Cortie, S.J.

1. The magnetic character of a day in the international scheme, inaugurated January 1906, is denoted by the figures 0 (quiet), 1 (moderately disturbed), and 2 (highly disturbed). The Stonyhurst lists, since 1889, give the character letters, *c* (calm), *s* (small), *m* (moderate), *g* (great), *vg* (very great), assigned by Father Sidgreaves, independently of any knowledge of the solar records. The international figure 2 would indicate a disturbance not less than the note *m*, which note would in addition include many disturbances of figure 1. A table of the sums of the disturbances *m*, *g*, *vg* has been drawn up for each year for the period 1889–1913. This period embraces three solar minima, 1889·6, 1901·7, 1913·4, and two solar maxima, 1894·1, 1906·4, according to Wolfer's tables. It covers two complete solar cycles. The total number of disturbances was 2028, of which 275 were of character *g*, and 29 of character *vg*.

2. With regard to sun-spots: (*a*) they rise rapidly in number and area to a maximum, and decline more slowly to a minimum. (*b*) A sun-spot cycle begins in higher mean latitudes and gradually falls in mean latitude to a minimum. Mr. Maunder's "butterfly wing" diagrams (*Monthly Notices, R.A.S.*, lxxiv. p. 114) illustrate this phenomenon. (*c*) Two cycles overlap, the high latitude spots of a second cycle occurring at the same time as the low latitude spots of its predecessor. (*d*) In general, in the ascending part of the curve of sun-spot variation until maximum is reached, the predominance of spots is in the northern hemisphere; after the maximum, the spots are more numerous in the southern hemisphere. These considerations are of importance in all investigations regarding the efficiency of sun-spots in relation to terrestrial magnetic storms. In individual sun-spot disturbances the character of the disturbance is also to be reckoned with in establishing such relations (*Monthly Notices, R.A.S.*, lxxiii. p. 153).

3. Magnetic disturbances are relatively more numerous when the sun-spot area is decreasing than when it is increasing.

(*a*) *Years of Decreasing Spot-area, including Minima.*

	All.	Great.	Very Great.
1st cycle, 1894–1901	637	41	15
2nd cycle, 1906–1913	691	147	12
1889	148	23	0
Totals	1476	211	27
Means	86·8	12·4	1·6 per year

(*b*) *Years of Increasing Spot-area, including Maxima.*

	All.	Great.	Very Great.
1st cycle, 1890–1893	428	59	0
2nd cycle, 1902–1905	124	5	2
Totals	552	64	2
Means	69·0	8·0	0·3

Again, in the 25 years 1889–1913 there were 7 years with a mean daily disc-area, or projected area, greater than 1000 units (one 10^{-6} of the sun's visible hemisphere), and 18 years with a mean daily disc-area less than 1000 units. Of the latter class 12 years were on the descending branch of the sun-spot curve. In the maximum years the mean yearly number of disturbances was 100, and the mean daily disc-area 1537·7 units. In the remaining years the numbers were 73·7 and 378·9. Defining the "efficiency-ratio" to be the ratio between the numbers expressing the mean yearly number of magnetic disturbances, and the mean daily disc-area of spots, the ratios are, for maximum years 0·065, for other years 0·195, a number three times as great as that derived for maximum sun-spot years.

4. The efficiency of sun-spots in relation to terrestrial magnetic disturbances depends on the positions of the spots. This is shown in the following table, in which the spots are arranged according to distance from the sun's equator, with their respective magnetic efficiency ratios.

Number of Years.	Mean Distance from Equator.	Mean Yearly Number Magnetic Disturbances.		Mean Daily Disc-Area.	Efficiency Ratios.	
	Spots.	All.	G. & V.G.	Spots.	All.	G. & V.G.
6	5°–10°	64·0	8·7	339·0	·189	·026
12	10°–15°	98·0	16·7	992·3	·099	·017
4	15°–20°	51·2	6·8	692·3	·074	·010
3	20°–25°	84·3	9·0	296·0	·285	·030

The decrease in the efficiency magnetically of spots with mean distance from the equator is well shown from 0° to 20°. But there is a seeming anomaly for the highest solar latitudes 20°–25° which needs explanation. In the first place, the means are derived from three years only, namely, 1890, 1891, 1913. Secondly, in these three years the two solar cycles overlapped, the high latitude

spots of a second cycle being contemporaneous with the low latitude spots of its predecessor. The anomaly, therefore, rather confirms than contradicts the results for other solar latitudes.

5. The reason why the position of a disturbed area on the sun is more important in magnetic efficiency than its size, is that after the maximum the mean latitude of the spots is falling towards the sun's equator, and since the heliographic latitude of the earth varies between $\pm 7°$, the earth is placed in a more favourable position to be affected by a solar disturbance. This would be particularly true of any disturbed solar area between the limits 0° and ± 10°. For a similar reason, as explained in a former communication (*Monthly Notices, R.A.S.*, lxxiii. p. 58), the minima in the curve of annual variation of magnetic storm frequency occur near the nodes of the sun's equator, and the maxima near points situated 90° from the nodes. For at and near the minima the heliographic latitude of the earth is situated in regions where there are comparatively few sun-spots, and at and near the maxima, in the regions between 5° and 10°, where, as the table shows, the magnetic efficiency is greatest. Similarly the predominance of spots in the southern hemisphere during a solar cycle is reflected in the amplitude of the two portions of the curve of annual variation of magnetic storms, which is much greater in the portion corresponding to the months December to June, from the descending to the ascending node, than in the portion from the ascending to the descending node. In the period December to June the heliograph latitude of the earth is south of the solar equator.

6. The results discussed above are derived from a consideration of the magnetic efficiency of sun-spots in general. They do not militate against the magnetic effects of particular large active spots, which as shown in a former paper (*Monthly Notices, R.A.S.*, lxxiii. p. 149) have been effective in latitudes removed from the solar equator as much as 18°. As stated before (*loc. cit.*, p. 153), "it would seem that a general state of sun-spot activity corresponds with a general state of terrestrial magnetic activity, but that it requires the advent of a large spot, the influence from which extends in all directions, or of a spot favourably situated in heliographic latitude relatively to the earth, to upset the equilibrium by the precipitation of a magnetic storm."

7. The conditions of great disturbance, and favourable position relatively to the earth, were both fulfilled in the spot-group which appeared 1915 June 12 to June 21. It was on the central meridian June 18. On that date, as also on June 16, the region of the sun containing this spot-group was greatly agitated, the granulations and the faculæ appearing to stream southwards. Spectroheliograms of the region received from Mount Wilson through the kindness of Professor Hale, and from Yerkes through that of Professor Frost, show extensive and brilliant H_α and calcium flocculi about the spot-group. The spot-group again was unique in that, with insignificant exceptions in 1913, it was the only spot-

group that has appeared so near the solar equator since the beginning of 1913. All other spot-disturbances of the present solar cycle had been confined to regions above 12° on each side of the sun's equator. On June 17 it extended in latitude from +0·0° to +2·7°, and in longitude from 37·4° to 31·2°. The heliographic latitude of the earth was +1·3°, in most favourable conjunction with the spot-group. A violent magnetic storm, the greatest so far recorded during the present solar cycle, took place on June 17, which was accompanied by fine displays of Aurora Borealis in North America, and of Aurora Australis in New Zealand.

Stonyhurst College Observatory:
1915 *November* 6.

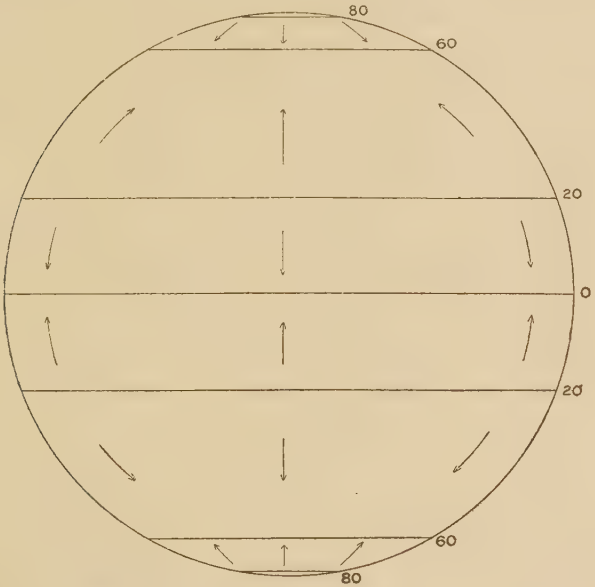
On Convection Currents in High Regions of the Solar Atmosphere.
By F. Henroteau, D. ès Sc. (Plates 1 and 2.)

(*Communicated by Rev. A. L. Cortie, S.J.*)

It has been recognised for a long time that there exists round the Sun a restricted layer disturbed by violent movements. Hence the study of the upper layers of the Sun seems to possess considerable importance in this regard. From his spectro-heliographic studies of these layers (cf. *Annales de l'Observatoire de Meudon*, iv. p. 96) M. Deslandres states that the filaments, which are only found in the high regions of the solar atmosphere, often group themselves on a curve which encircles the pole at distances varying between 50° and 70° of heliographic latitude, without, however, conforming itself to a parallel of latitude. These polar curves of filaments are in general more developed in the hemisphere, or on the meridians, where spots and faculæ are relatively feeble. They seem, indeed, to be more pronounced in years of minimum. Sometimes they are well marked in the neighbourhood of both poles, but more often they are in evidence about one pole only. Moreover, they seem to be displaced alternately from one pole to the other. Sometimes they assume an extraordinary development, showing points directed towards the equator, as if the matter of which they are formed was carried violently in that direction. According to M. Deslandres, who bases his conclusions upon researches made with the spectro-register of velocities, these filaments, where the solar material is rising, constitute zones of low pressure, while the spot zones, where faculæ are abundant and where the matter descends, form zones of high pressure. Whence the conclusion is drawn that there ought to exist convection currents from the spot zones towards the poles. Nevertheless, one can deduce even more from these observations of M. Deslandres with regard to the disposition of the currents.

During my daily observations of the solar surface I have often

remarked that the solar poles were covered with faculæ, a fact which when first observed struck me very forcibly. But when the distribution of the faculæ is studied more closely, it appears that on each hemisphere there are two maxima, one in the neighbourhood of 80°, and the other about 20°. A study of this distribution has been made by M. Mascari at the observatory of Catania (*Astrophysical Journal*, vi. p. 371). Taking account of this fact, we can deduce from it that the convection currents proceed in a general manner from the regions of faculæ towards those of the filaments, or rather towards the regions of the



Approximate distribution in latitude of convection currents in the Sun's higher regions.

minima of faculæ. These currents could be well represented by the above diagram.

An important verification of this deduction has been furnished by the researches of Mr. Slocum on solar prominences. From the examination of the appearance of 3323 prominences, photographed by means of the Rumford spectro-heliograph of the Yerkes Observatory, he draws the following conclusions:—(1) At mean latitudes, there is a tendency of movement towards the poles: in the northern hemisphere between the latitudes 15° and 53°, with a maximum of 69 per cent. at the latitude 33°, and in the southern hemisphere between 25° and 55°, with a maximum of 61 per cent. at latitude 52°. (2) In high latitudes there is a tendency for movement towards the equator: in the northern hemisphere with a maximum of 78 per cent. at latitude 63°, and in the southern

hemisphere with a maximum of 56 per cent. at the latitude 68°. (3) At the equator there is no marked tendency of movement, but there is a feeble movement towards the poles between 0° and −15°, and towards the equator between 0° and +15°. (4) The contrast between these two tendencies of movement is more accentuated for the northern than for the southern hemisphere, in the proportion of about 2 to 1. The results of Slocum, therefore, coincide approximately with the currents already indicated on the diagram.

The observations of faculæ during a period of great activity, such as we are witnessing at present, seem to indicate the existence of these main convection currents, either towards the poles or towards the equator. A more detailed examination of certain groups of Sun-spots, associated with large regions of faculæ, also leads us to deduce the existence of such currents, thus confirming the hypothesis of M. Deslandres, with which the observations of Slocum on prominences are in accord. The tendency of movement of faculous regions during periods of great activity has been very marked. On going through the series of drawings which I have made for the last year at this observatory, I find the following notes, which were written before undertaking any special study of the question, and independently of any theory.

1915 June 18, 9.45 G.M.T.—The spot regions are very disturbed. The granulations and the faculæ are visible at the centre of the disc, and seem to be produced towards the south of each group. This is to be remarked also with regard to the drawing of June 16. (Plate 1.)

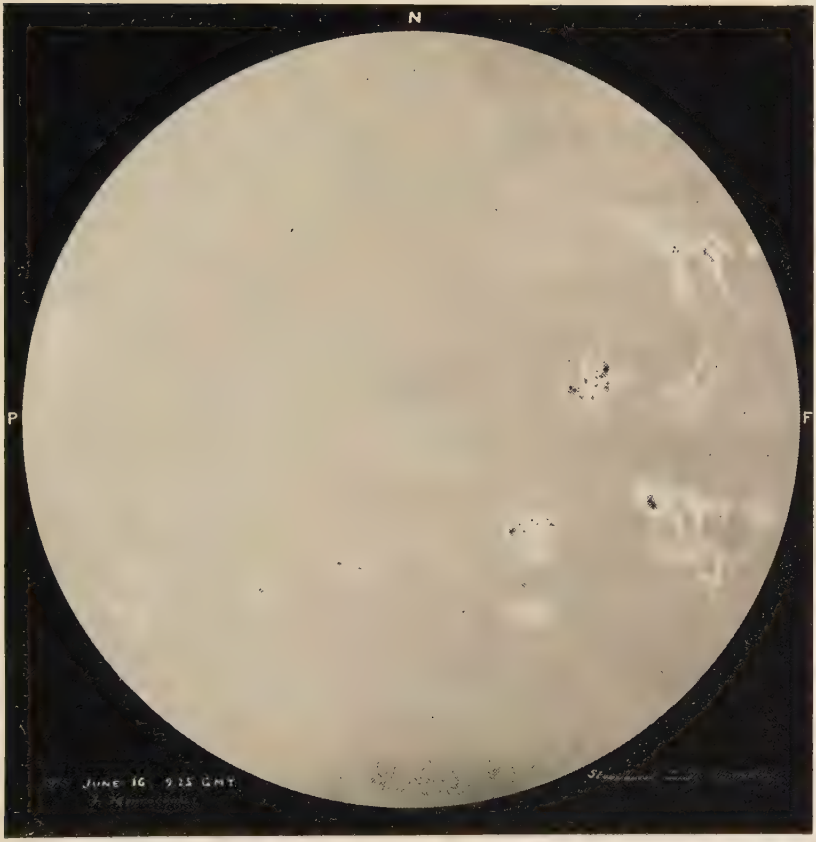
1915 June 19, 9.30 G.M.T.—Regions of faculæ are beautifully visible extending towards the north pole.

1915 July 26, 9.5 G.M.T.—The different groups in the southern hemisphere seem to be connected with faculæ and granulations, and as one can see on the drawing, these faculæ are stretching towards the pole of the Sun. (Plate 2.)

1915 August 14, 11.40 G.M.T.—The surface of the Sun seems to be very disturbed as a whole. Granulations and faculæ are visible in the disturbed regions, and what is to be noticed, as in different other cases of great solar activity, is that the faculæ seem to be directed towards the poles. This is to be remarked here of the southern hemisphere.

The drawing of June 16, of which a reproduction is given, shows the existence of these currents in the two hemispheres very well. The two currents of faculæ which in the N.E. quadrant are directed from the two groups of small spots, in latitudes +19° and +17°, towards the equator are particularly marked. All these phenomena, so far as the latitudes of the currents are concerned, seem to be perfectly in accord with the directions of the currents indicated on the diagram.

Sometimes the study of the groups of spots, associated with such regions of faculæ, seems to show the existence of these convection currents even better than the faculæ themselves. For



Drift of faculæ. N. hemisphere towards the equator. S. hemisphere towards the pole.

Drift of faculæ. N.W. quadrant towards the pole. S. hemisphere towards the pole.

this purpose let us take two examples of small groups of spots, nearly on the same meridian of longitude, but at different latitudes. We form the following tables :—

Date.	1st Group.		Area.	2nd Group.		Area.
	Lat.	Long.		Lat.	Long.	
Aug. 10	−29·7	342·1	8·7	..	..	...
11	−29·9	340·5	16·0	..	...	...
12	−30·2	340·7	16·9	...	...	...
13	−29·6	340·4	13·6	−20·0	340·0	37·4
14	−29·4	340·4	7·9	−20·0	340·0	20·9
15	−28·3	340·6	3·7	−20·0	340·0	13·4

From the time of the appearance of the second group, the tendency of the faculæ to flow towards the poles is shown perfectly, especially on August 14. The appearance of the second group on the same meridian as the first group, a phenomena which is relatively frequent, and the greater area of the second group seem to indicate that there is a movement of matter, not at the level of the spot-groups, since these maintain their positions, but at higher levels. The sequence of events would be as follows: first the appearance of the groups at latitude −29°, then of that at −20°, and the faculous matter spreading from the centres of activity towards the pole would tend to cover up the group at −29°, and cause it to disappear.

The second example seems even more striking than the first. On August 15 there appeared on the solar limb a faculous region which seemed to be clearly directed towards the equator. The faculæ in the southern hemisphere, we may remark in passing, showed an equally clear tendency to be directed towards the pole. At that date there were no spots in this faculous region, but after August 17 two groups of spots appeared on the same meridian. The following table shows their relative evolution :—

Date.	1st Group.		Area.	2nd Group.		Area.
	Lat.	Long.		Lat.	Long.	
Aug. 17	+4·8	279·9	36·7	+15·4	282·3	14·2
18	+4·0	281·7	32·4	+15·6	282·3	36·4
19	+4·0	282·0	13·8	+15·8	282·0	74·3
20	+4·0	282·0	11·9	+15·2	282·0	99·2
21	+3·7	284·4	9·3	+15·2	284·2	59·1
22	+3·8	283·8	1·2	+15·0	284·0	53·4
23	...	...	...	+14·2	288·8	51·6
24	...	...	...	+14·2	287·9	86·2
25	...	...	...	+15·0	284·0	95·1

When on August 25 the region of the two groups was near the west limb, the faculæ indicated clearly two currents: a movement

towards the equator, and to the north of the second group a movement towards the pole. Since, as the table shows, the area of the second group increases, while that of the first group diminishes, the faculous matter about the second group fills up the void caused by the vortex forming the spots of the first group, while that of the second group increases. The existence of a current seems to be beyond doubt, and the two tables are in exact accord with the directions of the currents indicated on the diagram.

Examples could be multiplied, but it is sufficient to have been able to give an indication which seems to be valid as to the existence of convection currents, as indicated by the flow of faculæ, in the higher regions of the Sun's atmosphere.

Stonyhurst Observatory:
1915 *November* 5.

Provisional Orbit Elements, Comet 1915 *a* (*Mellish*).

By C. J. Merfield.

From three observations by Dr. Baldwin for the dates 1915 April 12, April 20, and April 30, an approximate system of parabolic orbit elements of this comet was deduced. An observation by the same observer, for the date 1915 May 17, becoming available, it was used in combination with the first and last ones employed in the first determination.

The results of the final calculation indicate that these elements were no improvement upon those determined from the shorter arc, but they are used here as a basis for the more accurate determination.

It was at first thought that a considerable correction to the assumed eccentricity was necessary, but upon preparing the condition equations it was noted that small errors of observation would have considerable effect upon the time of perihelion passage and the angular distance between the node and perihelion.

The provisional orbit elements of this comet have been deduced from twelve equations of condition between the dates 1915 Feb. 25 and 1915 June 2.

The first and last positions were formed from the combination of several complete observations, and were given a weight of four; the remainder were single observations with weight unity.

The following elements form the basis of the work:—

$$\left.\begin{aligned} \omega &= 247^\circ\ 44'\ 11''\cdot6 \\ \Omega &= 72^\circ\ 16'\ 35''\cdot7 \\ \iota &= 54^\circ\ 46'\ 44''\cdot8 \end{aligned}\right\} 1915$$

$\tau = 1915$ July $17^d\cdot149852$, G.M.T.
$q = 1\cdot0058217$
Log $q = 0\cdot0025210$.

The equator was adopted as the fundamental plane of reference, the position of this approximate orbit in reference to this plane being

$$\left.\begin{array}{rrrr} \omega' = 272^\circ & 33' & 51''\cdot7 \\ \Omega' = 59 & 32 & 19\cdot2 \\ \iota' = 64 & 31 & 27\cdot7 \end{array}\right\} 1915.$$

We have also

$$\begin{aligned} d\omega_0 &= [9\cdot17112]d\Omega' - [9\cdot47192]d\iota'. \\ d\Omega &= [0\cdot00127]d\Omega' - [9\cdot71095]d\iota'. \\ d\iota &= [9\cdot57871]d\Omega' + [9\cdot95788]d\iota'. \\ d\omega &= d\omega' - d\omega_0. \end{aligned}$$

After the reduction of the condition to the normal equations the following corrections to the preceding elements were deduced :—

$$\begin{aligned} 10^4\, d\tau &= +53\cdot378 \\ d\omega' &= +126''\cdot02 \\ d\Omega' &= +\ 0''\cdot27 \\ d\iota' &= +\ 28''\cdot10 \\ 10^6\, dq &= -544\cdot193. \end{aligned}$$

We therefore have

$$\begin{aligned} d\omega &= +134''\cdot31 \\ d\Omega &= -\ 14\cdot17 \\ d\iota &= +\ 25\cdot61. \end{aligned}$$

Assuming the correction to the parabolic eccentricity to be unknown, we have the following

Provisional Orbit Elements, Comet 1915 *a* (Mellish).

$$\begin{aligned} \tau &= 1915 \text{ July } 17\cdot155190 - [6\cdot40583]de, \text{ G.M.T.} \\ \left.\begin{array}{rrrr} \omega = 247^\circ & 46' & 25''\cdot9 & +1\cdot033de \\ \Omega = 72^\circ & 16' & 21''\cdot5 & +0\cdot139de \\ \iota = 54^\circ & 47' & 10''\cdot4 & +0\cdot375de \end{array}\right\} & 1915 \\ q &= 1\cdot0052775 + [4\cdot33565]de \\ \text{Log } q &= 0\cdot0022860 \\ e &= 1\cdot0 + [5\cdot00000]de, \end{aligned}$$

the value of de to be expressed in units of the fifth decimal place.

The value of de determined from the present calculation is 3·5743, giving a value for e as 1·0000357, which is so near to unity that it has been eliminated and the orbit elements given in terms of de, so that any value may be adopted at pleasure.

Co-ordinates.

$$\begin{aligned} x &= [9\cdot7979610]r \sin(\ 38^\circ\ 46'\ 15''\cdot97 + v) \\ y &= [9\cdot9489556]r \sin(348\ 24\ 24\cdot69 + v) \\ z &= [9\cdot9556047]r \sin(272\ 35\ 57\cdot80 + v) \end{aligned}$$

A comparison of the calculated with the observed places gives the following residuals :—

1915.	Cos δda.	$d\delta$.
	s	"
Feb. 26	−0·02	−0·2
Apr. 12	+0·25	+4·7
,, 20	−0·06	+1·6
,, 30	+0·21	−2·6
May 17	−0·01	0·0
June 1	−0·01	+0·1

The same values are obtained by introducing the values of the unknown quantities, determined from the normal equations, into the condition equations.

Melbourne Observatory :
1915 *July* 1.

Micrometrical Measures of 400 *Double Stars.* By W. S. Franks.

The following measures were made with the $6\frac{1}{8}$-inch Cooke refractor at Mr. Frederick J. Hanbury's observatory, Brockhurst, East Grinstead. The list of Struve stars, with their co-ordinates, is taken from Lewis' revision of the *Mensuræ Micrometricæ* (*Memoirs, R.A.S.*, vol. lvi.). In the selection, I gave preference to the usually neglected wider pairs, where measures were few and not very recent; also with the object of furnishing data for proper motions. Two eyepieces were used, of magnifying power 235 and 345 diameters, the latter being reserved for the closer pairs. Bright field illumination was necessarily used, in the absence of any provision for bright wires and dark field. Three measures of angle and four of distance were usually made, and on two or more nights; one revolution of the screw = 25″·38. The magnitudes are as taken from the catalogue, except where I occasionally met with discrepancies, and made a note of such—prefixing my own estimates by Fr. In the Σ *rej.* pairs the magnitudes of the *comites* are often overrated. In the columns headed Pos. Ang., Distance, and Epoch, the *mean* values of the separate measures are given—though not pretending to any greater accuracy than the first decimal place in the position angles.

Abbreviations used. — A, angle; D, distance; <, increasing; >, decreasing; cols., colours; orge., orange; yel., yellow; ylsh., yellowish; grnsh., greenish; bl., blue; blsh., bluish; prp., purple; prpsh., purplish; wh., white; v., very; p., pale; o, no colour seen.

Measures of Double Stars.

Σ.	R.A. 1900. h m	Decl. 1900. ° ′	Magnitudes.	Pos. Ang. °	Distance. ″	Epoch.	Nights.	Remarks.
058	0 0·0	+29 46	7·7, 9·2	52·75	13·99	1914·649	2	A and D <.
9	,, 8·5	49 0	8·8, 9·0	165·9	20·01	·672	2	
10	,, 9·4	62 14	7·5, 8·2	175·7	17·57	·669	2	
12	,, 9·8	8 16	6·2, 7·8	149·55	11·29	·680	2	35 Piscium ; ylsh. wh., purp.
17	,, 11·3	28 44	8·0, 9·2	29·4	26·81	·672	2	
20	,, 12·2	15 57	8·0, 9·0	232·4	11·54	·669	2	
28	,, 18·5	28 59	7·9, 8·1	223·1	33·09	·669	2	
30	,, 21·8	49 26	6·8, 8·7	302·35	17·64	·680	2	A < ; D >.
36	,, 27·2	6 24	6·2, 9·3	82·55	27·60	·674	2	51 Piscium ; wh., blsh.
38	,, 29·8	58 7	8·3, 8·7	144·55	17·45	·680	2	
39	,, 29·4	− 5 6	6·8, 8·5	43·4	19·29	·831	2	P. O 113 ; A and D > ; ylsh. wh., blsh.
40	,, 29·8	+36 16	6·8, 8·8	313·6	11·60	·678	2	
3 *rej.*	,, 38·3	− 1 26	Fr. 8·0,9·0	342·7	29·47	·831	2	A and D < ; *comes* rated 8·7 mag.
62	,, 44·8	+35 16	8·5, 9·2	303·05	11·83	·678	2	
72	,, 49·1	38 38	8·0, 9·0	177·05	23·98	·678	2	A >.
80	,, 54·3	0 16	7·0, 8·2	323·3	22·36	·831	2	P. O 251 ; A and D < ; p. orge., bl.
84	,, 58·6	0 50	6·6, 9·0	252·5	16·27	·831	2	26 Ceti ; wh., blsh.
86	,, 59·7	− 6 1	8·0, 8·7	153·9	13·50	·831	2	A > ; D <.
*88	1 0·3	+20 56	4·9, 5·0	159·85	29·67	·689	2	ψ^1 Piscium ; wh., blsh. wh.
90	,, 0·7	4 22	5·9, 6·8	82·8	32·67	·831	2	77 Piscium ; ylsh. wh., ruddy.
2 *rej.*	,, 3·1	44 40	Fr. 8·0, 10·0	358·65	15·58	·689	2	*Comes* rated 9·0 mag.
93	,, 22·6	88 47	2·5, 9·0	216·15	18·24	·683	2	α Ursæ Min. ; ylsh. wh., bl.
94	,, 5·1	16 4	8·7, 8·7	275·35	19·44	·874	2	
98	,, 7·3	31 33	7·0, 8·0	248·4	19·70	·874	2	
100	,, 8·5	7 3	4·2, 5·3	63·45	23·76	·870	2	ζ Piscium ; ylsh. wh , lilac.
112	,, 14·8	45 50	8·5, 9·0	327·3	22·81	·874	2	
131	,, 26·8	60 10	6·0, 9·2	142·7	13·80	·877	2	Yel., blsh.
136	,, 29·6	12 3	6·9, 8·0	78·3	15·74	·870	2	100 Piscium ; wh., purpsh.
139	,, 33·0	52 28	8·8, 9·0	223·45	10·05	·874	2	
146	,, 36·0	9 36	8·3, 8·3	306·65	24·42	·870	2	D < ?
150	,, 38·4	− 7 35	7·2, 7·8	195·55	36·61	·870	2	
163	,, 44·1	+64 21	6·2, 8·2	34·3	34·99	·877	2	Ruddy orge., bl. ; colours remarkable.
171	,, 43·7	− 1 55	8·4, 8·5	160·7	31·11	·874	2	A and D <.
177	,, 46·2	+ 4 27	8·5, 9·0	121·85	32·90	·874	2	D >.
199	,, 57·4	67 12	8·5, 8·5	21·55	36·00	·877	2	
206	1 57·5	+10 54	8·0, 9·2	134·35	31·50	1914·877	2	

* 88. An 11·0 mag. (H) at 107°·7 : 73″·85 from B.

Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch	Nights.	Remarks.
	h m	° ′		°	″			
222	2 4·8	+38 34	6·7, 7·2	35·4	16·50	1914·898	2	59 Androm.; wh., lilac.
230	,, 7·8	58 1	7·5, 8·7	259·65	24·32	·915	2	A < ?
231	,, 7·7	− 2 52	6·0, 7·8	232·9	15·54	·898	2	66 Ceti. A < ? p. yel., blue.
239	,, 11·6	+28 17	6·0, 7·5	211·0	14·11	·898	2	P. II 38–9. A < ? ylsh. wh. lilac.
242 *rej.*	,, 11·3	−10 17	Fr. 7·0, 9·5	235·9	56·74	·886	2	A and D <; *comes* rated 9 mag.
245	,, 11·8	+39 50	7·0, 8·0	292·15	10·46	·904	2	
246	,, 12·7	34 2	7·8, 8·5	122·35	10·20	·904	2	
265	,, 19·4	− 2 12	8·2, 8·7	135·7	11·95	·898	2	
274	,, 26·4	+ 0 39	7·0, 7·5	218·5	13·52	·898	2	
289	,, 34·8	26 38	5·8, 8·7	360·85	28·80	·915	2	33 Arietis; wh., bl.
292	,, 36·2	39 50	7·5, 8·2	210·9	23·06	·917	2	
307	,, 43·4	55 29	4·0, 8·5	300·6	28·33	·917	2	η Persei. Orge., bl., fine colour
328	,, 51·1	44 7	8·5, 9·0	300·75	22·05	·917	2	D > ?
329	,, 53·5	58 38	7·5, 9·0	273·65	16·24	·922	2	
331	,, 53·8	51 57	5·3, 6·7	84·65	11·82	·922	2	P. II 220; wh., bl.
335	,, 56·4	63 22	8·0, 8·5	160·15	23·26	·922	2	A <; D >.
337	,, 55·8	41 0	7·5, 9·0	162·75	17·25	·922	2	
338	,, 56·3	10 27	8·2, 8·5	200·7	20·08	·917	2	
359 *rej.*	3 5·5	22 2	7·5, 8·5	321·45	32·58	·928	2	A <; D >.
*370	,, 10·4	32 16	Fr. 8·4, 10·0	315·85	17·04	·928	2	A < ?
374	,, 15·0	67 6	7·0, 8·5	295·5	11·03	·928	2	
379	,, 16·8	29 27	8·5, 8·5	101·05	10·75	·928	2	
390	,, 22·3	55 6	4·8, 9·2	159·1	14·78	·928	2	P. III 57; ylsh. wh., blsh.
396	,, 24·8	58 26	6·3, 8·0	242·85	20·24	·936	2	Wh., bl.
399 *rej.*	,, 24·7	19 46	Fr. 8·0, 9·5	145·4	20·27	·936	2	A > ? *comes* rated 9·0 mag.
401	,, 25·3	27 14	6·5, 7·0	270·05	11·32	·936	2	Both wh.
426	,, 34·2	38 48	7·0, 8·5	340·85	20·14	·936	2	D < ?
†431	,, 36·0	33 39	Fr. 5·0, 9·5	237·5	19·86	·966	3	40 Persei; ylsh. wh., o.
435	,, 37·1	25 22	7·3, 7·8	1·1	13·07	1915·000	2	
436	,, 36·1	−12 56	7·0, 8·1	236·55	37·91	·010	2	A and D <.
447	,, 41·5	+38 2	7·8, 9·0	168·55	28·02	·010	2	A >; D <.
451 *rej.*	,, 41·0	−13 40	8·0, 8·5	321·95	20·12	·010	2	Lewis gives decl. 13° 46′.
455	,, 47·0	+69 13	8·2, 8·7	165·05	12·08	·000	2	
474	,, 59·5	75 59	8·5, 8·5	147·0	23·29	·006	2	A and D < ?
476	,, 55·0	38 23	7·5, 8·7	287·7	21·63	·010	2	A and D <.
510	4 6·9	+ 0 29	6·5, 9·5	302·8	11·38	1915·015	2	Slight < in A and D.

* 370. Σ mags. 8·2, 10·3; but Lewis 8·5, 8·7—*qu. var.* ?
† 431. Erroneously called *o* Persei, and rated 4·2 mag.

Measures of Double Stars—continued.

Σ.	R.A. 1900. h m	Decl. 1900. ° ′	Magnitudes.	Pos. Ang. °	Distance. ″	Epoch.	Nights.	Remarks.
518	4 10·7	− 7 47	4·0, 9·2	105·0	82·39	1915·010	2	40 Eridani ; yel., bl.
519 *rej.*	,, 13·4	+50 8	7·5, 9·0	346·3	18·05	·020	2	Lewis has decl. 40° 8′. D > ?
528	,, 16·5	25 24	5·7, 8·8	24·05	19·28	·020	2	χ Tauri ; wh., lilac.
533	,, 17·9	34 5	6·3, 7·5	61·05	19·23	·020	2	Wh., blsh.
534	,, 18·0	24 4	6·0, 8·0	290·05	28·97	·020	2	62 Tauri ; ylsh. wh., lilac.
548	,, 22·6	30 8	6·0, 8·0	35·55	14·34	·020	2	Ylsh. wh., bl.
550	,, 24·1	53 41	5·1, 6·2	306·85	10·34	·020	2	1 Cam. ; ylsh. wh., prp.
551	,, 24·5	51 59	8·5, 9·0	126·45	13·52	·020	2	
557	,, 28·4	62 46	8·0, 8·7	125·6	23·47	020	2	
587	,, 40·1	52 56	7·2, 8·3	185·05	20·78	·029	2	
599	,, 43·8	44 47	8·0, 9·3	335·4	10·33	·029	2	
602	,, 48·2	69 9	8·3, 9·5	134·45	29·12	·029	2	
606	,, 49·5	69 16	8·0, 8·8	299·35	37·62	·029	2	
623	,, 53·7	27 10	6·8, 8·3	204·2	20·46	·029	2	Wh., bl.
624	,, 52·8	− 5 55	8·3, 8·8	88·85	28·65	·029	2	
627	,, 55·3	+ 3 28	6·3, 7·0	258·4	20·61	·029	2	Slight > in A and D.
630	,, 56·8	1 28	6·8, 8·0	49·25	14·24	·029	2	P. IV 278–9 ; wh., prpsh.
642 *rej.*	5 1·8	− 4 47	6·0, 9·2	9·8	52·98	·037	2	66 Eridani ; wh., o. *Comes* 9·5 Fr.
649	,, 3·6	− 8 47	7·0, 8·7	75·2	21·67	·048	2	A >.
650 *rej.*	,, 4·6	+13 53	8·0, 8·1	165·8	26·68	·037	2	
671	,, 11·2	25 58	8·5, 9·0	124·75	17·57	·045	2	
681	,, 13·2	46 52	6·0, 8·5	180·65	23·29	·045	2	P. yel., blsh.
691	,, 16·6	31 4	8·5, 9·0	300·1	25·25	·045	2	
696	,, 17·6	3 27	5·0, 7·0	28·15	32·02	·045	2	23 Orionis ; ylsh. wh., bl.
698	,, 18·6	34 46	6·2, 7·7	346·6	31·03	·045	2	P. yel., bl.
704	,, 23·0	69 35	7·2, 9·5	13·6	20·52	·068	2	A < ; D >.
737	,, 29·8	34 4	8·1, 8·5	304·9	10·75	·053	2	In Cluster M 36.
752	,, 30·6	− 5 59	3·7, 7·7	141·75	11·45	·053	2	ι Orionis ; grnsh. wh., bl.
753	,, 32·2	+30 25	5·6, 8·0	267·95	12·37	·053	2	26 Aurigæ ; p. yel., prp.
759	,, 33·5	17 40	8·5, 9·1	324·0	29·66	·053	2	
764	,, 35·0	29 26	6·3, 6·8	13·7	25·92	·068	2	Wh., p. bl.
775	,, 37·4	40 21	8·1, 8·9	66·65	22·94	·068	2	D < ?
781	,, 38·7	32 21	Fr. 8·7, 9·7	121·55	14·81	·083	2	Lewis mgs. 7·4, 8·4 ; Σ 8·7, 10·2.
834	,, 58·1	30 14	Fr. 8·0, 8·5	308·35	22·87	·068	2	,, ,, 8·3, 8·3 ; Σ 8·0, 8·8 : at least ½ mg. difference.
838	6 0·0	0 52	6·7, 8·8	326·0	39·74	·083	2	Yel., blsh.
855	6 3·7	+ 2 31	5·8, 6·8	113·5	29·31	1915·083	2	3rd star 8·8 : 106°·5 : 117″·64 : est. 8·0 in 1883, but 10·0 in 1911.

Measures of Double Stars—continued.

Σ	R.A. 1900. h m	Decl. 1900. ° ′	Magnitudes.	Pos. Ang. °	Distance. ″	Epoch.	Nights.	Remarks.
861	6 4·9	+30 42	7·8, 8·2	17·9	64·37	1915·086	2	A – BC. A < ; D >.
866	,, 8·9	62 14	7·7, 8·8	194·65	17·99	·083	2	AB } Triple.
,,	,, ,,	,, ,,	,, 8·2	265·6	79·25	·083	2	AC }
872	,, 8·9	36 12	6·0, 7·0	217·2	10·90	·086	2	P. yel., lilac.
889	,, 13·7	25 2	7·2, 9·5	230·6	21·54	·092	2	A <.
900	,, 18·5	4 39	4·0, 6·7	25·75	13·74	·092	2	8 Monoc.; ylsh. wh., lilac.
911	,, 21·7	4 8	8·1, 8·1	155·95	12·97	·092	2	Small > in A and D.
921	,, 25·6	11 19	6·0, 8·2	3·85	16·30	·092	2	P. yel., bl.
933	,, 29·7	41 13	8·0, 8·5	74·7	25·69	·095	2	
943	,, 31·7	23 16	8·5, 9·0	142·9	22·36	·095	2	D <.
962	,, 41·9	26 49	8·5, 8·5	241·7	25·70	·095	2	
973	,, 50·8	75 22	6·6, 7·6	27·15	11·86	·092	2	Ylsh. wh., lilac.
994	,, 52·6	37 15	7·2, 7·5	56·45	26·05	·095	2	
1000	,, 53·3	25 23	7·7, 8·7	66·5	21·51	·095	2	D >?
1004	,, 53·8	–11 17	8·0, 9·1	87·1	19·95	·092	2	D <.
1006	,, 57·6	+62 42	7·0, 8·0	72·0	29·66	·101	2	D >.
1023	7 2·4	25 10	8·0, 8·5	102·8	24·68	·108	2	
1044	,, 9·1	47 53	8·5, 8·7	168·05	12·18	·108	2	
1050	,, 11·7	55 7	7·3, 8·0	19·2	19·47	·108	2	
1054	,, 11·5	35 8	7·3, 8·5	291·6	18·70	·108	2	
1062	,, 14·7	55 28	5·3, 6·6	315·0	14·88	·108	2	19 Lyncis ; wh., lilac.
1065	,, 14·6	50 20	6·6, 6·8	253·45	15·20	·108	2	20 ,, wh., wh.
1067	,, 13·6	3 4	7·7, 8·7	265·3	25·44	·119	2	
1086	,, 21·5	42 58	7·5, 9·0	103·1	12·27	·119	2	
1088	,, 20·4	14 18	7·0, 9·0	194·6	11·13	·119	2	
1090	,, 20·7	18 43	7·0, 8·0	97·3	60·37	·119	2	AB. Small > in D } Triple.
,,	,, ,,	,, ,,	8·0, 9·5	322·05	20·15	·119	2	BC. A and D < }
1091	,, 22·3	50 10	8·2, 8·7	328·45	29·44	·128	2	A > ; D <.
1108	,, 26·8	23 7	6·7, 8·5	178·9	11·42	·128	2	Lewis mags. 7·3, 8·9; Fr. as Σ.
1122	,, 36·4	65 24	7·1, 7·1	4·8	15·35	·135	2	P. ,, VII 159; 6·8, 7·6; both p. yel. do. }
1124	,, 34·9	22 4	8·2, 8·4	325·55	19·32	·135	2	
1129	,, 35·9	18 17	8·2, 8·7	62·75	21·70	·137	2	
1132	,, 37·2	– 3 17	Fr. 8·0, 8·4	234·8	19·89	·137	2	A >? ; Σ mags. 8·1, 8·7.
1149	,, 44·3	+ 3 29	7·3, 9·0	40·6	21·70	·137	2	
1169	8 1·1	79 48	7·6, 7·9	10·65	20·87	·161	2	
1183	,, 1·6	– 8 57	5·5, 7·8	326·45	30·71	·165	2	Wh., bl.
1184	,, 2·8	+38 13	8·0, 8·5	340·75	26·87	·165	2	
1198	8 6·1	+ 1 34	8·0, 8·2	157·15	33·37	1915·170	2	

Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nights.	Remarks.
	h m	° ′		°	″			
1217	8 17·3	+45 17	7·2, 8·7	241·15	29·05	1915·182	2	D > slowly ?
1227	,, 21·6	23 29	7·5, 8·8	163·35	24·49	·182	2	
1245	,, 30·5	6 58	6·0, 7·0	25·25	10·40	·196	2	P. VIII 108; p. yel., prp.
1255	,, 34·4	6 8	7·0, 8·0	31·2	26·77	·196	2	
1268	,, 40·6	29 8	4·4, 6·5	307·3	30·42	·196	2	ι Cancri; yel., bl. (vivid cols.).
1276	,, 41·8	11 31	7·9, 8·1	354·35	12·40	·196	2	
1283	,, 44·3	15 12	7·0, 7·5	123·3	16·56	·196	2	
1293	,, 52·1	54 22	7·8, 9·0	92·5	18·59	·196	2	
1315	9 4·8	62 5	7·0, 7·2	25·7	24·81	·209	2	
1326	,, 14·2	78 52	7·7, 8·1	171·6	29·35	·213	2	
1347	,, 18·1	3 56	6·7, 8·0	310·6	21·35	·213	2	P. IX 64–5; ylsh. wh., bl.
1349	,, 22·7	67 59	6·8, 8·0	164·9	19·40	·213	2	Ylsh. wh., blsh. wh.
1350	,, 24·2	67 14	7·2, 7·3	246·45	10·52	·213	2	AB = 0Σ 203 } Triple.
,,	,, ,,	,, ,,	7·3, 8·0	211·6	121·57	·213	2	BC. A < ? }
1351	,, 23·6	63 30	3·8, 9·0	271·8	22·90	·213	2	23 Ursæ Maj.; ylsh. wh., prpsh.
1358	,, 24·5	45 7	Fr. 7·7, 8·8	161·85	23·19	·219	2	Σ 7·3, but only about 1 mag. diff. now—*qu. var.* ? A < D >.
1360	,, 25·2	11 3	7·4, 7·7	62·9	14·33	·213	2	
1369	,, 29·1	40 25	7·0, 8·0	147·3	24·78	·219	2	AB } Trple.
,,	,, ,,	,, ,,	,, (Fr.) 8·0	324·0	118·24	·219	2	AC; not noted by Σ }
1399	,, 51·5	20 13	6·8, 7·8	174·9	30·29	·219	2	Wh., blsh. wh.
1415	10 9·8	71 34	6·1, 7·0	166·9	16·72	·224	2	Ylsh. wh., blsh. wh.
1442	,, 26·5	22 33	7·2, 7·8	155·55	13·23	·224	2	= 0Σ 221.
1448	,, 28·9	22 6	7·0, 9·0	259·25	10·72	·224	2	
1449	,, 29·4	35 39	8·5, 8·7	288·0	36·08	·228	2	
1458	,, 33·8	32 13	8·0, 8·2	216·1	17·67	·228	2	
1477	,, 44·4	13 28	8·3, 8·8	275·05	17·45	·228	2	
1482	,, 47·0	8 1	8·0, 8·9	305·25	11·79	·224	2	P. X 179.
1486	,, 49·1	52 39	7·5, 8·8	102·25	28·93	·234	2	
1495	,, 53·7	59 27	6·0, 8·3	38·05	34·42	·234	2	P. yel., bl.
1502	,, 56·8	15 9	8·5, 9·3	282·05	13·07	·228	2	D < ?
1520	11 10·3	53 19	6·5, 7·8	344·4	12·85	·234	2	P. yel., prp.
1533	,, 16·7	37 38	8·2, 8·4	172·6	23·22	·242	2	
1540	,, 21·7	3 33	6·3, 7·3	149·9	29·22	·242	2	83 Leonis; ylsh. wh., ruddy.
1544	,, 25·6	60 15	7·0, 8·0	90·3	12·40	·242	2	
1547	,, 26·6	14 55	6·4, 8·4	322·6	15·35	·242	2	88 Leonis; p. yel., lilac.
1561	,, 33·5	45 40	5·9, 8·0	257·0	9·96	·264	2	AB. A >; p. yel., blsh. wh. } Triple.
,,	,, ,,	,, ,,	,, 8·0	88·95	111·58	·264	2	AC. Rapid < in D }
1565	11 34·4	+19 33	7·0, 8·0	304·2	21·60	1915·260	2	

Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nights.	Remarks.
	h m	° ′		°	″			
1573	11 43·7	+67 53	6·6, 7·6	177·65	11·38	1915·260	2	Wh., blsh.
1575	,, 46·8	9 22	7·0, 8·0	209·45	30·61	·260	2	
1588	,, 57·2	72 55	8·5, 8·7	50·95	14·56	·264	2	A and D >.
1603	12 3·1	56 1	6·9, 7·3	81·5	22·38	·276	2	Both wh.
1615	,, 9·1	33 20	6·0, 8·4	88·35	26·69	·276	2	Yel., prpsh.
1622	,, 11·1	41 12	5·7, 8·0	259·5	11·48	·276	2	2 Can. Ven. ; orge., bl.
1625	,, 11·9	80 41	6·5, 7·0	218·75	14·37	·419	2	Wh., p. wh.
1627	,, 13·0	− 3 24	5·9, 6·4	196·9	19·98	·264	2	P. XII 32–3 ; Lewis 5·7, 6·2, bu Fr. as Σ ; ylsh. wh., blsh. wh
1636	,, 17·4	+ 5 52	6·2, 9·0	336·6	20·02	·283	2	17 Virginis ; ylsh. wh., blsh.
1657	,, 30·1	18 56	4·7, 6·2	271·0	20·30	·283	2	24 Comæ Ber. ; yel., blsh. lilac
1677	,, 40·1	− 3 20	7·0, 8·0	347·7	15·76	·283	2	Lewis 6·8, 7·2 ; Fr. as Σ.
1682	,, 46·1	− 9 47	6·7, 9·0	304·95	31·60	·286	2	P. XII 196; A and D >; yel., blsh
1685	,, 47·0	+19 42	6·8, 7·3	201·35	16·02	·283	2	P. XII 201–2 ; both ylsh. wh.
1689	,, 50·5	12 2	6·7, 9·0	209·45	28·68	·286	2	P. XII 221 ; Lewis decl. − 7° 58″ p. orge., bl. ; A <.
1692	,, 51·3	38 51	3·2, 5·7	226·65	19·96	·286	2	α Can. Ven. ; ylsh. wh., fawn or p. ruddy.
1694	,, 48·3	83 57	4·9, 5·4	326·3	21·57	·419	2	Ylsh. wh., lilac.
1702	,, 53·8	38 50	8·0, 8·5	82·35	35·53	·283	2	
1732	13 8·6	58 58	8·0, 9·5	126·95	26·56	·295	2	
1740	,, 18·5	3 14	7·1, 7·2	75·7	27·12	·302	2	
1744	,, 19·9	55 27	2·1, 4·2	150·0	14·56	·304	2	ζ Ursæ Maj. ; wh., grnsh. wh.
1756	,, 28·6	23 32	8·5, 9·0	177·15	14·18	·308	2	
1764	,, 32·6	2 54	7·0, 8·7	31·65	15·67	·308	2	
1766	,, 32·6	30 36	Fr. 8.5, 9·0	67·95	19·98	·308	2	Σ 8·3; 9·3.
1782	,, 40·3	18 52	7·7, 9·2	185·2	29·60	·308	2	
1786	,, 45·2	35 29	8·0, 9·5	22·9	10·53	·408	2	
1797	,, 57·2	19 54	8·2, 8·5	159·65	21·03	·308	2	
1803	14 2·2	38 54	7·7, 9·5	42·9	17·73	·408	2	
1821	,, 9·9	52 16	4·5, 6·6	237·0	12·81	·316	2	κ Boötis ; ylsh. wh., prp.
1838	,, 19·2	11 42	7·2, 7·3	333·55	9·30	·450	2	
1839	,, 18·1	54 21	8·3, 8·3	79·55	14·34	·316	2	A >?
1840	,, 18·0	68 14	6·5, 9·2	221·9	27·60	·316	2	Ylsh. wh., blsh.
1843	,, 21·0	48 18	7·2, 8·7	187·4	20·05	·316	2	
1850	,, 24·1	28 44	6·1, 6·7	261·9	25·41	·316	2	Ylsh. wh., blsh. wh.
1855	,, 28·1	32 5	8·2, 9·1	248·9	15·20	·316	2	
1869	,, 37·4	− 5 32	8·0, 9·0	132·15	25·60	·408	2	
1872	,, 38·2	+58 24	7·0, 8·0	41·25	7·67	·450	2	
1874	14 38·7	+49 33	7·7, 9·2	287·85	26·72	1915·319	2	D <?

Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nights.	Remarks.
	h m	° ′		°	″			
1882	14 41·5	+61 33	7·2, 8·7	0·9	11·35	1915·319	2	
1895	,, 53·7	40 34	7·8, 8·3	43·5	12·50	·319	2	
1901	,, 56·8	31 46	7·8, 9·5	196·4	25·0	·319	2	A and D >.
1902	,, 57·2	16 10	8·0, 8·5	187·1	26·11	·319	2	
1904	,, 59·1	5 53	7·0, 7·0	346·7	9·76	·450	2	
1909	15 0·5	48 3	5·2, 6·1	247·83	4·07	·452	3	44 Boötis ; yel., prpsh.
1910	,, 2·7	9 37	7·0, 7·0	212·7	4·45	·452	3	P. XIV 279. A <.
1921	,, 8·2	39 2	7·0, 7·2	283·4	30·52	·321	2	
1927	,, 9·9	62 13	7·1, 8·0	353·45	16·06	·321	2	
1931	,, 13·9	10 47	6·2, 7·6	169·5	13·23	·321	2	A > sml. ; p. yel., blsh.
3093	,, 17·5	− 1 10	8·0, 9·2	142·1	28·87	·480	2	A <; D >.
1952	,, 27·1	+10 0	7·8, 9·0	221·55	15·83	·450	2	
1954	,, 30·0	10 53	3·2, 4·1	188·2	3·96	·452	3	δ Serp. ; wh., grnsh. yel.
1962	,, 33·2	− 8 28	6·3, 6·4	187·75	11·78	·324	2	Both yel.
1963	,, 33·8	+30 26	7·3, 7·7	292·95	4·74	·450	2	
1965	,, 35·6	36 58	4·9, 5·9	304·33	6·32	·452	3	ζ Cor. ; wh., grnsh. bl.
1970	,, 41·6	15 44	3·0, 9·2	265·05	30·71	·321	2	β Serp. ; ylsh. wh., lilac.
1972	,, 35·0	80 47	6·1, 7·0	81·45	30·79	·419	2	π′ U. Min.; yelsh. wh., blsh. wh.
1973	,, 42·6	36 45	7·3, 8·5	319·95	30·48	·324	2	A > ?
1976	,, 43·0	59 44	8·2, 8·2	71·65	19·50	·324	2	D < ?
1984	,, 48·5	53 11	6·2, 8·5	275·45	6·40	·450	2	Wh., blsh.
1985	,, 50·7	− 1 52	7·0, 8·1	339·29	5·93	·450	2	A <.
1986	,, 50·7	+10 23	8·2, 8·8	93·6	14·39	·485	2	
1987	,, 52·2	3 41	7·2, 8·7	323·4	10·26	·450	2	P. XV 220.
1995	,, 55·8	14 55	8·3, 9·3	313·25	15·33	·324	2	A <; D >.
1999	,, 58·4	−11 10	7·4, 8·1	99·45	11·32	·380	2	A >; D <; nr. ξ Scorp.
2010	16 3·6	+17 19	5·0, 6·0	11·40	29·41	·347	2	κ Herc.; p. yel., dull orge.; A <; D >.
2018 *rej.*	,, 8·3	− 7 23	8·4, 9·1	356·4	19·19	·380	2	D >.
2021	,, 8·6	+13 48	6·7, 6·9	337·7	3·75	·391	3	49 Serp.; both ylsh. wh.; A <.
2029	,, 9·8	29 0	7·5, 9·3	186·6	6·37	·450	2	
2044	,, 20·6	37 16	7·8, 8·0	342·6	8·53	·389	3	
2051	,, 24·7	10 49	7·1, 8·6	18·5	13·57	·354	2	
2056	,, 26·7	5 39	7·9, 9·0	316·05	6·18	·450	2	
2063	,, 28·8	45 49	5·7, 8·2	194·95	16·43	·354	2	Wh., prpsh.
2065	,, 29·3	40 13	8·0, 8·7	216·65	31·12	·354	2	D < ?
2078	,, 33·8	53 7	5·0, 6·0	110·07	3·64	·389	3	17 Drac.; p. yel., lilac.
2079	,, 35·3	23 11	7·1, 7·9	90·7	16·66	·354	2	
2085	16 38·1	+21 46	7·3, 8·3	307·8	6·19	1915·394	3	

Measures of Double Stars—continued.

Σ.	R.A. 1900. h m	Decl. 1900. ° ′	Magnitudes.	Pos. Ang. °	Distance. ″	Epoch.	Nights.	Remarks.
2092	16 37·7	+60 53	7·7, 8·8	3·8	8·26	1915·389	3	
2095	,, 41·1	28 32	7·0, 9·0	162·7	5·13	·394	3	46 Herc.; ylsh. wh., blsh.
2096	,, 42·1	2 14	6·0, 9·3	90·75	23·16	·360	2	19 Oph.; wh., blsh.; D <?
2098	,, 41·8	30 13	8·0, 9·0	146·0	14·31	·354	2	AB
,,	,, ,,	,, ,,	,, 8·0	135·15	65·84	·354	2	AC. D< } Quad.
,,	,, ,,	,, ,,	,, Fr.9·7	16·6	63·68	·354	2	AD
2104	,, 45·1	36 6	6·2, 8·0	18·7	5·73	·394	3	Wh., lilac.
2105	,, 46·3	1 19	8·0, 9·5	130·1	29·11	·480	2	
2116	,, 54·4	63 40	8·2, 8·8	4·3	19·70	·360	2	A >; D <.
2120	17 0·8	28 14	6·4, 9·2	238·4	8·64	·456	2	A >; D <; p. yel., v. bl
2122	,, 1·7	− 1 31	6·5, 8·7	280·05	20·52	·380	2	Ylsh. wh., blsh.
2123	,, 2·1	+ 6 55	8·5, 8·5	217·5	18·87	·380	2	
2131	,, 5·8	30 28	7·5, 8·5	177·8	24·25	·374	2	A >.
2135	,, 7·9	21 21	7·1, 8·4	181·0	7·67	·454	2	A and D <.
2138	,, 8·1	54 37	8·0, 8·3	134·8	22·33	·374	2	A >.
2145	,, 12·6	26 41	8·0, 9·5	181·5	14·06	·374	2	AC. A and D <.
2159	,, 20·3	13 25	7·4, 8·1	326·45	26·63	·380	2	
2164	,, 21·0	47 21	7·8, 9·3	9·8	9·83	·466	2	A >; D <.
2165	,, 22·4	29 32	7·0, 8·5	54·65	8·42	·456	2	A and D <. AB } Trip.
,,	,, ,,	,, ,,	,, Fr.9·5	251·4	97·46	Rated 8·8m, but quite 1m < B.		AC }
2166	,, 23·2	11 28	5·6, 7·4	282·85	27·16	·380	2	Wh., p. bl.
2178	,, 25·8	35 0	7·0, 8·6	129·75	10·59	·380	2	
2191	,, 34·5	− 4 55	7·0, 8·0	267·8	26·43	·454	2	
2194	,, 37·0	+24 32	6·2, 8·5	8·9	16·12	·380	2	P. XVII 200; yel., lilac.
2202	,, 39·5	2 37	5·5, 5·8	93·2	20·47	·380	2	61 Oph.; both wh.
2204	,, 40·7	−13 16	7·0, 7·2	24·4	14·28	·485	2	
2219	,, 40·3	+61 39	8·0, 9·0	99·5	17·56	·466	2	A > slowly.
2235	,, 47·7	− 2 14	7·5, 9·1	124·15	19·25	·462	2	D <?
2237	,, 46·8	+41 59	7·2, 9·5	8·1	20·30	·485	2	
2259	,, 55·2	30 3	7·0, 8·0	278·0	19·54	·380	2	
2264	,, 57·3	21 35	4·9, 4·9	259·65	6·35	·462	2	95 Herc.; grnsh. wh., ruddy wh.
2268	,, 59·2	25 22	8·0, 9·0	209·85	21·05	·466	2	AC. A >; D <.
2273	,, 58·7	64 8	6·8, 7·3	284·4	20·81	·380	2	Ylsh. wh., blsh. wh.
2277	18 0·6	48 28	6·3, 8·2	121·35	27·09	1914·697	2	A <; D >; wh., bl.
2276	,, 1·1	12 0	Fr. 6·5,7·0	257·45	6·74	1915·466	2	Σ 6·0, 7·0; but only $\frac{1}{2}$m diff. P. XVII 362; ylsh. wh., blsh. wh.
2280	,, 3·8	26 5	5·9, 5·9	2·4	13·80	1914·697	2	100 Herc.; wh., flushed wh.
2308	18 7·5	+79 59	5·4, 6·1	233·2	19·75	1914·697	2	40, 41 Drac.; both p. yel. A > slowly.

Measures of Double Stars—continued.

Σ	R.A. 1900. h m	Decl. 1900. ° ′	Magnitudes.	Pos. Ang. °	Distance. ″	Epoch.	Nights.	Remarks.
2325	18 25·8	−10 52	6·0, 9·2	256·95	12·21	1915·527	2	Wh., blsh.
2340	,, 29·3	+31 31	8·3, 9·2	103·4	23·57	1914·702	2	D <.
2348	,, 31·8	52 13	5·9, 8·1	272·35	25·79	·705	2	Yel., bl.
2354*rej.*	,, 33·4	38 37	Fr. 8·5, 9·5	302·45	29·47	·702	2	*sp* Vega ; *comes* rated 9·0 mg.
2372	,, 38·6	34 39	6·7, 8·2	83·2	25·00	·705	2	Wh., bl.
2374	,, 39·5	27 37	8·8, 9·2	38·0	14·72	·702	2	A < ; D >.
2376	,, 39·9	30 18	7·7, 8·4	63·9	22·27	1915·537	2	
2379	,, 41·3	− 1 5	5·6, 7·4	121·05	13·10	1914·707	2	AB. 5 Aquil. ; ylsh. wh., lilac.
2380	,, 40·0	+44 49	6·7, 8·2	9·65	25·98	·705	2	P. yel., bl.
2391	,, 43·3	− 6 8	6·2, 9·0	332·45	37·80	1915·527	2	Wh., blsh.
2417	,, 51·2	+ 4 4	4·0, 4·2	102·95	22·17	1914·716	2	$\theta^1\theta^2$ Serp. ; both p. yel.
2420	,, 49·7	59 17	4·6, 7·6	332·2	32·45	·712	2	*o* Drac. ; orge. yel., bl. ; A > ; D <.
2424	,, 54·5	13 29	5·7, 9·2	270·9	16·15	·707	2	11 Aquil. ; ylsh. wh., bl. ; A < ; D >.
2425	,, 55·1	− 8 15	6·8, 7·7	180·1	31·38	1915·530	2	Yel., p. bl. ; A and D >.
2426	,, 55·3	+12 44	6·5, 8·2	259·35	16·74	·537	2	P. orge., blsh. grey.
2431	,, 55·5	40 32	6·3, 9·2	236·6	18·87	1914·707	2	Wh., bl.
2434	,, 57·6	− 0 51	7·4, 7·9	119·95	23·00	·712	2	AB. P. XVIII 274–5 ; A and D >.
2436	,, 57·3	+ 8 37	7·4, 8·1	311·85	32·27	·712	2	A < ; D >.
2439	,, 59·6	− 7 18	8·0, 9·0	199·25	22·07	1915·537	2	
2440	,, 56·3	+62 15	6·2, 9·0	123·1	17·38	1914·702	2	P. yel., bl. ; D <.
2442	,, 59·2	16 49	8·0, 9·5	207·4	17·38	·702	2	D >.
2445	19 0·4	23 10	6·3, 8·0	261·2	12·17	·719	2	Wh., bl.
2446	,, 0·9	6 24	6·3, 8·3	153·7	9·17	1915·542	2	P. XVIII 302 ; wh., bl.
2447	,, 1·4	− 1 30	6·7, 9·1	344·65	14·04	·542	2	Ylsh. wh., blsh.
2449	,, 1·5	+ 6 59	7·1, 7·8	291·15	0·10	·542	2	
2457	,, 2·9	22 25	7·2, 8·7	200·55	10·12	·542	2	Lewis mgs. 8·2, 8·7.
2458	,, 2·8	27 36	8·5, 9·0	227·0	11·89	1914·716	2	D <.
2470	,, 5·1	34 36	6·7, 8·2	269·45	13·38	·719	2	A > ; wh., blsh. wh.
2472	,, 5·1	37 45	7·5, 9·2	337·3	19·14	1915·577	2	AB. P. XIX 13 ; D < } Quad.
,,	,, ,,	,, ,,	,, 9·0	348·55	74·90	·577	2	AC. Σ 2472–3 } Quad.
2473	,, ,,	,, ,,	9·0, 9·2	292·6	6·14	·577	2	CD.
2474	,, 5·4	34 27	6·7, 8·0	259·3	16·87	1914·719	2	P. yel., bl.
2483	,, 8·5	30 11	7·2, 8·3	315·8	10·16	1915·577	2	AB. A > ; D < } Triple.
,,	,, ,,	,, ,,	,, 8·5	236·5	71·30	·577	2	AC } Triple.
2487	,, 10·4	38 59	4·1, 8·0	81·9	28·20	1914·721	2	η Lyræ ; wh., bl. ; A >.
2497	19 15·1	+ 5 24	6·9, 8·0	357·15	30·24	1914·724	2	Yel., blsh. wh.

Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nights.	Remarks.
	h m	° ′		°	″			
2498	19 15·2	+ 3 51	7·2, 7·8	66·1	12·05	1915·585	2	
2501	,, 16·8	− 4 56	7·3, 8·8	22·0	19·96	1914·724	2	
2504	,, 16·6	+18 57	6·4, 8·1	284·8	8·77	1915·585	2	A > s; ylsh. wh., blsh.
2507	,, 16·7	44 11	8·2, 9·2	140·2	26·03	1914·719	2	A and D <.
2519	,, 22·7	− 9 44	8·0, 8·1	123·3	11·33	1915·577	2	
2532	,, 25·1	+ 2 41	6·0, 10·0	5·2	33·83	·589	3	P. XIX 144; p. orge., grnsh.; D > sml.
2533	,, 24·9	− 0 39	7·2, 9·0	211·3	22·56	1914·724	2	
2537	,, 28·4	− 4 24	8·3, 8·7	133·35	19·07	·724	2	
2542	,, 29·5	+52 46	8·2, 8·7	254·15	11·60	·726	2	
2547	,, 33·4	−10 34	7·7, 9·0	331·25	20·70	1915·577	2	
2554	,, 32·4	+60 3	7·9, 8·4	194·9	18·65	1914·726	2	A >.
2562	,, 37·9	8 9	6·5, 8·2	252·9	27·06	·726	2	P. XIX 241; ylsh. wh., lilac.
2567	,, 39·4	12 7	7·5, 9·2	314·2	18·29	1915·585	2	P. XIX 250.
2571	,, 34·2	78 2	7·3, 8·0	20·85	11·30	·590	2	A > sml.
2573	,, 38·7	60 17	6·2, 8·5	26·85	18·13	·590	2	Wh., bl.; A > sml.
2578	,, 42·0	35 50	6·6, 7·4	124·7	14·78	1914·726	2	P. XIX 276–7; wh., blsh. wh.; Lewis mgs. 6·2, 6·8.
2580	,, 42·6	33 30	5·1, 8·1	71·3	25·87	·730	2	χ Cyg.; p. yel., lilac.
2591	,, 48·0	− 6 16	7·5, 8·5	106·6	29·25	1915·577	2	A >.
2604	,, 51·7	+63 54	6·5, 8·7	184·4	27·83	1914·726	2	Yel., bl.
2636	20 6·4	− 4 53	8·2, 9·2	203·6	12·40	·729	2	
2649	,, 8·6	+31 47	7·7, 8·8	151·6	23·35	·730	2	D >.
2654	,, 9·9	− 3 49	6·2, 7·7	233·85	14·02	·730	2	Ylsh. wh., bl.
2660	,, 10·7	+64 13	8·2, 9·0	167·3	22·10	1915·601	2	
2661	,, 14·7	− 2 34	7·5, 8·7	342·5	24·40	1914·730	2	
2664	,, 14·9	+12 42	7·7, 8·2	322·15	27·92	1915·601	2	
2679	,, 19·8	19 14	7·4, 8·7	80·0	23·06	1914·732	2	D <.
2680	,, 20·2	14 32	8·3, 8·5	287·75	16·36	·730	2	
2687	,, 24·0	56 19	6·5, 8·3	117·35	26·33	·737	2	Wh., blsh.
2690	,, 26·4	10 55	7·0, 7·5	256·3	16·30	·734	2	P. XX 178–7.
2691	,, 26·0	37 48	8·0, 8·2	32·5	17·13	1915·601	2	
2692	, 26·7	26 8	8·0, 9·0	301·0	25·68	·601	2	
2693	,, 25·7	54 10	8·0, 9·0	13·5	13·73	·601	2	
2700	,, 30·7	32 9	6·5, 8·3	285·45	23·92	1914·737	2	Ylsh. wh., bl.
2703	,, 32·2	14 23	7·6, 7·6	289·9	25·31	·737	2	AB
,,	,, ,,	,, ,,	,, 7·6	235·7	72·30	·737	2	AC. C moving } Triple.
2707	,, 34·6	47 35	7·1, 8·6	30·75	22·11	·737	2	AB. Mags. of B and C transposed in Lewis } Triple.
,,	20 ,,	+ ,, ,,	,, 7·9	195·55	55·40	1914·737	2	AC

Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nights.	Remarks.
	h m	° ′		°	″			
2727	20 42·0	+15 46	4·0, 5·0	269·9	10·94	1914·740	2	γ Del.; golden yel., grnsh. bl.
2733	,, 47·8	6 57	8·0, 8·3	145·25	40·04	·740	2	P. XX 355–6.
2738	,, 53·9	16 3	7·2, 8·2	254·95	14·56	·737	2	
2743	,, 56·4	47 7	4·7, 9·0	352·7	20·40	·740	2	59 Cyg. AB ; wh., bl.
2750	21 0·3	12 19	7·8, 9·3	279·8	17·04	·752	2	D <.
2769	,, 6·0	22 2	6 5, 7·5	299·95	17·93	·752	2	Wh., blsh.
2779	,, 10·1	28 40	8·5, 8·5	179·65	16·53	·752	2	A and D >.
2787	,, 16·7	1 35	Fr. 7·5,8·5	19·8	22·78	1915·601	2	Lewis 8·0, 9·0 ; Σ 7·0, 8·3.
2793	,, 20·2	8 57	7·0, 8·7	241·85	26·57	·601	2	
2796	,, 17·6	78 11	7·3, 8·8	43·6	25·47	1914·760	2	D <.
2806	,, 27·4	70 7	3·0, 8·0	250·55	13·52	·760	2	β Ceph.; grnsh. wh., prp.
2809	,, 32·4	− 0 50	6·5, 8·5	163·15	31·50	·751	2	Σ 6·0 mag. ; wh., bl.
2810	,, 31·6	+58 40	7·5, 8·5	290·25	17·03	·818	2	
2819	,, 37·3	57 7	7·5, 8·5	57·35	12·40	·818	2	P. XXI 256.
2829	,, 45·0	30 17	8·2, 8·9	15·65	17·44	·831	2	D <?
2832	,, 45·6	50 3	7·8, 8·3	212·4	13·03	·818	2	
2838	,, 49·3	− 3 47	6·0, 8·8	182·55	18·71	·752	2	A and D >; ylsh., blsh.
2840	,, 48·6	+55 19	6·0, 7·0	194·65	18·84	·818	2	D >; wh., prp.
2841	,, 49·6	19 14	6·5, 8·0	111·25	22·20	·752	2	Yel., bl.
2848	,, 53·0	5 28	7·2, 7·5	56·35	10·46	·752	2	
2857	22 1·3	9 35	7·0, 8·7	112·95	19·79	·834	2	
2865	,, 2·2	69 43	8·5, 9·0	191·2	21·54	·834	2	A and D <.
2867	,, 5·1	7 26	7·9, 9·0	208·5	10·50	·836	2	
2876	,, 7·6	37 10	7·7, 9·2	68·35	11·82	·836	2	
2883	,, 8·3	69 38	6·2, 8·2	253·3	14·55	·833	2	Ylsh. wh., blsh.
2886	,, 10·8	48 51	7·4, 9·5	108·6	20·08	·842	2	Slow < D.
2893	,, 11·1	72 49	5·5, 7·6	348·5	28·90	·836	2	Ylsh. wh., bl.
2894	,, 14·5	37 16	6·0, 8·2	193·15	15·41	·836	2	P. XXII 65 ; ylsh. wh., grey.
2896	,, 15·2	62 43	7·5, 8·5	241·7	20·61	·842	2	D >.
2915	,, 27·5	6 54	8·5, 8·7	148·4	12·72	·844	2	A >.
2920	,, 29·4	3 41	7·1, 8·2	143·6	13·74	·842	2	P. XXII 145.
2943	,, 42·4	−14 35	6·0, 9·2	116·9	25·70	·836	2	τ^1 Aquarii ; A< ; D >; wh., prp.
2985	23 5·3	+47 25	7·0, 8·0	253·5	15·33	·850	2	
2986	,, 5·0	13 53	6·5, 9·3	271·0	31·47	·850	2	A >; wh., o.
2992	,, 8·3	39 27	7·5, 9·2	285·8	14·34	·850	2	Slow < D.
2998	,, 13·8	−14 0	5·2, 7·2	346·9	13·68	·850	2	94 Aquarii; yel., bl., good cols.
3019	,, 25·6	+ 4 42	7·1, 8·1	184·95	10·68	·850	2	
3028	23 33·6	+34 29	7·0, 9·5	202·95	17·03	1914·862	2	A and D >.

Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos Ang.	Distance.	Epoch.	Nights.	Remarks.
	h m	° ′		°	″			
3031	23 36·0	+ 5 43	7·5, 8·5	312·0	14·30	1914·850	2	
3043	,, 47·8	38 8	8·4, 9·2	250·7	16·06	·862	2	Slow < D.
3044	,, 47·9	11 22	6·9, 7·3	282·05	19·18	·855	2	P. XXIII 216–7; slow < D; ylsh., blsh.
3052	,, 57·8	70 48	7·2, 7·8	7·4	34·48	·862	2	Slow < D.
3053	,, 57·5	65 33	6·0, 7·3	71·5	15·30	·862	2	A <; yel., bl.
3054	23 57·9	+ 7 43	7·5, 8·5	180·9	33·73	1914·855	2	

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Measures of Southern Binary Stars. By John Tebbutt.

The following are the results of measures of α Centauri and γ Coronæ Australis with the Grubb 8-inch equatorial and Cooke filar-micrometer.

Star.	Date.	Position Angle.	Distance.	No. of Evenings.
		°	″	
α Centauri	1913·457	217·0	18·41	10
γ Coronæ Aust.	1913·646	100·2	2·28	5
α Centauri	1915·415	219·1	17·66	10

The final results are derived by weighting the results for each evening according to the number of measures and the conditions for observing. The measures of α Centauri were taken in full sunlight, and those of γ Coronæ Australis in twilight. In consequence of a considerable failure of my eyesight since 1913, I regard the measures of 1915, particularly those of distance, as not so satisfactory as the results of 1913.

Observatory, Peninsula, Windsor,
N.S. Wales:
1915 *October* 2.

The Dynamics of a Stellar System. Third Paper: Oblate and other Distributions. By A. S. Eddington, M.A., M.Sc., F.R.S., Plumian Professor.

1. In a former paper * I have treated of a stellar system, having spherical symmetry, in a steady state of motion, and with a distribution of velocities according to Schwarzschild's ellipsoidal law. An exact solution of this problem was obtained, so that it was possible to specify the density and law of velocities at all parts of a system of this kind. In the present paper the restriction of spherical symmetry is removed, and the investigation is extended to oblate and other systems.

In the previous work the system was supposed to move under forces arising solely from its own attraction; but we shall now find it useful to keep distinct those results which are true when the system moves under any conservative system of forces, and the additional limitations imposed when those forces are identified with its own gravitational attractions. It is possible that the latter are not the only forces concerned; for example, L. V. King † has pointed out that H. S. Jones's value of the absorption of light leads to the conclusion that the quantity of residual gas in interstellar space is such as to produce a gravitational field very much greater than that of the whole system of lucid stars. Further, by postponing the assumption that there are no other forces, we can obtain results applicable to part of the system only, *e.g.* the stars of one particular mass; in this way we can better take into account the heterogeneity of the actual stellar system. Thus, except where expressly stated, we shall not introduce the limitation that the attraction of the system is the sole force concerned; and it will be seen that results of wide generality are obtained, which may be regarded as purely kinematical consequences of the ellipsoidal law of velocities.

The objects of the investigation are twofold:—

(1) We must determine the velocity-law at all parts of the system. It is *assumed* that in any small region the frequency of a velocity (u, v, w) is proportional to $e^{-a^2u^2-b^2v^2-c^2w^2}$ conformably to the ellipsoidal law, where the exponent defines an ellipsoid known as the velocity-ellipsoid; but the velocity-ellipsoid will in general vary in size, shape, and orientation from one part of the system to another, and we wish to determine these variations. The problem is to fit together velocity-ellipsoids varying from point to point of space in such a way as to give a steady state of motion.

(2) We must also determine the density, or number of stars per unit volume, at all parts of the system.

From the various possible solutions we have then to pick out

* The Second Paper, *M.N.*, vol. lxxv. p. 366.

† *Nature*, vol. xcv. p. 701.

those which conform to certain facts derived from observation. The chief facts are:—

(1) In the neighbourhood of the Sun (which may be taken to be on the galactic plane at an unknown distance from the centre) the ratios of the axes of the velocity-ellipsoid are approximately 2 : 1 : 1.

(2) The density-distribution is very oblate. At the same distance from the centre towards the galactic poles and in the galactic plane respectively, the ratio of the densities is about 1 : 4 (average value).

The mathematical discussion depends on the Equation of Continuity, which expresses that the stars at any place have travelled in from neighbouring parts in the right numbers and with the right velocities to give the density and velocity distributions which exist there. The analysis employed differs from that of the previous paper, because when there is not spherical symmetry the equations of motion cannot be integrated; in some respects the new treatment will appear simpler, because it follows more closely the usual hydrodynamical methods.

As in my former work, the stars are supposed to move undisturbed by the chance arrangement of their immediate neighbours.*

Derivation of the Fundamental Equations.

2. At any point of the system the directions of the axes of the velocity-ellipsoid determine three directions at right angles. The velocity-ellipsoids thus define three orthogonal families of curves, each curve being traced by moving step by step always in the direction of an axis of the velocity-ellipsoid at the point reached. These curves may be regarded as the intersections of a triply orthogonal family of surfaces, which we shall call the *principal velocity-surfaces*. The axes of the velocity-ellipsoid at any point are normals to the three principal velocity-surfaces through that point.

We shall define our co-ordinates with reference to these surfaces, and take parameters λ, μ, ν of the three principal velocity-surfaces through any point as the curvilinear co-ordinates of that point.

Let the elements of length in the principal directions be

$$P d\lambda, \quad Q d\mu, \quad R d\nu$$

where P, Q, R are in general functions of λ, μ, ν.

Consider the motion of a particle under a gravitational field of potential $\phi(\lambda, \mu, \nu)$. We have, for the kinetic and potential energies,

$$T = \tfrac{1}{2}(P^2\lambda'^2 + Q^2\mu'^2 + R^2\nu'^2)$$
$$V = -\phi$$

where λ' stands for $d\lambda/dt$.

* *Stellar Movements and the Structure of the Universe*, pp. 247–255.

Then Lagrange's equations give

$$\frac{d}{dt}(\mathrm{P}^2\lambda') = \frac{1}{2}\left(\frac{\partial \mathrm{P}^2}{\partial\lambda}\lambda'^2 + \frac{\partial \mathrm{Q}^2}{\partial\lambda}\mu'^2 + \frac{\partial \mathrm{R}^2}{\partial\lambda}\nu'^2\right) + \frac{\partial\phi}{\partial\lambda}, \text{ etc.}$$

But
$$\frac{d\mathrm{P}^2}{dt} = \frac{\partial \mathrm{P}^2}{\partial\lambda}\lambda' + \frac{\partial \mathrm{P}^2}{\partial\mu}\mu' + \frac{\partial \mathrm{P}^2}{\partial\nu}\nu'.$$

Hence

$$\left.\begin{aligned}
\mathrm{P}^2\lambda'' &= -\frac{1}{2}\frac{\partial \mathrm{P}^2}{\partial\lambda}\lambda'^2 + \frac{1}{2}\frac{\partial \mathrm{Q}^2}{\partial\lambda}\mu'^2 + \frac{1}{2}\frac{\partial \mathrm{R}^2}{\partial\lambda}\nu'^2 - \frac{\partial \mathrm{P}^2}{\partial\mu}\lambda'\mu' - \frac{\partial \mathrm{P}^2}{\partial\nu}\lambda'\nu' + \frac{\partial\phi}{\partial\lambda} \\
\mathrm{Q}^2\mu'' &= \frac{1}{2}\frac{\partial \mathrm{P}^2}{\partial\mu}\lambda'^2 - \frac{1}{2}\frac{\partial \mathrm{Q}^2}{\partial\mu}\mu'^2 + \frac{1}{2}\frac{\partial \mathrm{R}^2}{\partial\mu}\nu'^2 - \frac{\partial \mathrm{Q}^2}{\partial\lambda}\lambda'\mu' - \frac{\partial \mathrm{Q}^2}{\partial\nu}\mu'\nu' + \frac{\partial\phi}{\partial\mu} \\
\mathrm{R}^2\nu'' &= \frac{1}{2}\frac{\partial \mathrm{P}^2}{\partial\nu}\lambda'^2 + \frac{1}{2}\frac{\partial \mathrm{Q}^2}{\partial\nu}\mu'^2 - \frac{1}{2}\frac{\partial \mathrm{R}^2}{\partial\nu}\nu'^2 - \frac{\partial \mathrm{R}^2}{\partial\lambda}\lambda'\nu' - \frac{\partial \mathrm{R}^2}{\partial\mu}\mu'\nu' + \frac{\partial\phi}{\partial\nu}
\end{aligned}\right\} \quad (1)$$

These are the equations of motion.

3. According to the ellipsoidal law, the number of stars in an element of volume $dxdydz$ with velocities between u, v, w and $u+du$, $v+dv$, $w+dw$, is

$$\mathrm{N} = \sigma dxdydz\,\pi^{-\frac{3}{2}}abce^{-a^2u^2-b^2v^2-c^2w^2}dudvdw$$

where σ is the density and a^{-1}, b^{-1}, c^{-1} are the semiaxes of the velocity-ellipsoid; the axes of co-ordinates are taken along the principal axes of the ellipsoid.

In curvilinear co-ordinates, since $u = \mathrm{P}\lambda'$, etc., this becomes

$$\mathrm{N} = \pi^{-\frac{3}{2}}abc\sigma \mathrm{P}d\lambda \mathrm{Q}d\mu \mathrm{R}d\nu \mathrm{P}d\lambda' \mathrm{Q}d\mu' \mathrm{R}d\nu' \exp \{-a^2\mathrm{P}^2\lambda'^2 - b^2\mathrm{Q}^2\mu'^2 - c^2\mathrm{R}^2\nu'^2\} \quad (2)$$

$$\log \mathrm{N} = \log(abc\sigma) + \log(\mathrm{P}^2\mathrm{Q}^2\mathrm{R}^2 d\lambda d\mu d\nu d\lambda' d\mu' d\nu') - a^2\mathrm{P}^2\lambda'^2 - b^2\mathrm{Q}^2\mu'^2 - c^2\mathrm{R}^2\nu'^2 - \tfrac{3}{2}\log\pi \quad (3)$$

Now the equation of continuity is *

$$\frac{\mathrm{D}\mathrm{N}}{\mathrm{D}t} = 0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which merely expresses that if we follow up a particular group of stars the number of them is constant, *i.e.* no stars are created or destroyed.

First consider $\dfrac{\mathrm{D}}{\mathrm{D}t}(d\lambda d\mu d\nu d\lambda' d\mu' d\nu')$.

* Stokes's operator, $\dfrac{\mathrm{D}}{\mathrm{D}t} \equiv \dfrac{\partial}{\partial t} + \lambda'\dfrac{\partial}{\partial\lambda} + \mu'\dfrac{\partial}{\partial\mu} + \nu'\dfrac{\partial}{\partial\nu} + \lambda''\dfrac{\partial}{\partial\lambda'} + \mu''\dfrac{\partial}{\partial\mu'} + \nu''\dfrac{\partial}{\partial\nu'}$ represents the rate of change of a property associated with a particular group of particles; $\dfrac{\partial}{\partial t}$, which represents the change of a property associated with a point of space, is always zero in our work, since the motion is steady. The functions a, b, c, P, Q, R, σ do not contain λ', μ', ν', so that for them $\dfrac{\mathrm{D}}{\mathrm{D}t} = \lambda'\dfrac{\partial}{\partial\lambda} + \mu'\dfrac{\partial}{\partial\mu} + \nu'\dfrac{\partial}{\partial\nu}$.

If in a time δt the co-ordinates of a star change from λ to λ_1, etc.,

$$\left.\begin{aligned}\lambda_1 &= \lambda + \lambda'\,\delta t \\ \lambda'_1 &= \lambda' + \lambda''\,\delta t,\end{aligned}\right\} \quad \text{neglecting squares of } \delta t \qquad (5)$$

Thus the Jacobian

$$J = \frac{\partial(\lambda_1\mu_1\nu_1\lambda_1'\mu_1'\nu_1')}{\partial(\lambda\mu\nu\lambda'\mu'\nu')}$$

becomes by (5)

$$\begin{vmatrix} 1 & 0 & 0 & \delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \delta t \\ \frac{\partial\lambda''}{\partial\lambda}\delta t & \frac{\partial\lambda''}{\partial\mu}\delta t & \frac{\partial\lambda''}{\partial\nu}\delta t & 1+\frac{\partial\lambda''}{\partial\lambda'}\delta t & \frac{\partial\lambda'}{\partial\mu'}\delta t & \frac{\partial\lambda''}{\partial\nu'}\delta t \\ \frac{\partial\mu''}{\partial\lambda}\delta t & \frac{\partial\mu''}{\partial\mu}\delta t & \frac{\partial\mu''}{\partial\nu}\delta t & \frac{\partial\mu''}{\partial\lambda'}\delta t & 1+\frac{\partial\mu''}{\partial\mu'}\delta t & \frac{\partial\mu''}{\partial\nu'}\delta t \\ \frac{\partial\nu''}{\partial\lambda}\delta t & \frac{\partial\nu''}{\partial\mu}\delta t & \frac{\partial\nu''}{\partial\nu}\delta t & \frac{\partial\nu''}{\partial\lambda'}\delta t & \frac{\partial\nu''}{\partial\mu'}\delta t & 1+\frac{\partial\nu''}{\partial\nu'}\delta t \end{vmatrix}$$

To the first order in δt the determinant reduces to its leading diagonal, so that the Jacobian becomes

$$J = 1 + \delta t\left(\frac{\partial\lambda''}{\partial\lambda'} + \frac{\partial\mu''}{\partial\mu'} + \frac{\partial\nu''}{\partial\nu'}\right).$$

Now $d\lambda_1 d\mu_1 d\nu_1 d\lambda'_1 d\mu'_1 d\nu'_1 = J\, d\lambda d\mu d\nu d\lambda' d\mu' d\nu'$,

and by definition it $= \left(1 + \delta t\,.\,\frac{D}{Dt}\right) d\lambda d\mu d\nu d\lambda' d\mu' d\nu'.$

Hence

$$\left(\frac{\partial\lambda''}{\partial\lambda'} + \frac{\partial\mu''}{\partial\mu'} + \frac{\partial\nu''}{\partial\nu'}\right) d\lambda d\mu d\nu d\lambda' d\mu' d\nu' = \frac{D}{Dt}\,(d\lambda d\mu d\nu d\lambda' d\mu' d\nu'),$$

and therefore

$$\frac{D}{Dt}\log\,(d\lambda d\mu d\nu d\lambda' d\mu' d\nu') = \frac{\partial\lambda''}{\partial\lambda'} + \frac{\partial\mu''}{\partial\mu'} + \frac{\partial\nu''}{\partial\nu'}\,. \qquad (6)$$

From (1)

$$\frac{\partial\lambda''}{\partial\lambda'} = \frac{1}{P^2}\left(-\frac{\partial P^2}{\partial\lambda}\lambda' - \frac{\partial P^2}{\partial\mu}\mu' - \frac{\partial P^2}{\partial\nu}\nu'\right)$$

$$= -\frac{1}{P^2}\,\frac{DP^2}{Dt} = -\frac{D}{Dt}\log P^2.$$

Similarly

$$\frac{\partial\mu''}{\partial\mu'} = -\frac{D}{Dt}\log Q^2\,; \quad \frac{\partial\nu''}{\partial\nu'} = -\frac{D}{Dt}\log R^2\,.$$

Hence (6) gives

$$\frac{D}{Dt}\log\,(P^2Q^2R^2 d\lambda d\mu d\nu d\lambda' d\mu' d\nu') = 0 \qquad (7)$$

Returning to equations (3) and (4), we obtain in consequence of (7)

$$\frac{\mathrm{D}}{\mathrm{D}t}\log(abc\sigma)=\frac{\mathrm{D}}{\mathrm{D}t}(a^2\mathrm{P}^2\lambda'^2+b^2\mathrm{Q}^2\mu'^2+c^2\mathrm{R}^2\nu'^2) \quad . \quad (8)$$

or, setting $abc\sigma=e^\kappa$ . . . . . (8a)

$$\begin{aligned}\frac{\partial\kappa}{\partial\lambda}\lambda'+\frac{\partial\kappa}{\partial\mu}\mu'+\frac{\partial\kappa}{\partial\nu}\nu'=&\frac{\partial(a^2\mathrm{P}^2)}{\partial\lambda}\lambda'^3+\frac{\partial(a^2\mathrm{P}^2)}{\partial\mu}\lambda'^2\mu'+\frac{\partial(a^2\mathrm{P}^2)}{\partial\nu}\lambda'^2\nu'\\&+\frac{\partial(b^2\mathrm{Q}^2)}{\partial\lambda}\lambda'\mu'^2+\frac{\partial(b^2\mathrm{Q}^2)}{\partial\mu}\mu'^3+\frac{\partial(b^2\mathrm{Q}^2)}{\partial\nu}\mu'^2\nu'\\&+\frac{\partial(c^2\mathrm{R}^2)}{\partial\lambda}\lambda'\nu'^2+\frac{\partial(c^2\mathrm{R}^2)}{\partial\mu}\mu'\nu'^2+\frac{\partial(c^2\mathrm{R}^2)}{\partial\nu}\nu'^3\\&+2a^2\Big(-\tfrac{1}{2}\frac{\partial\mathrm{P}^2}{\partial\lambda}\lambda'^3+\tfrac{1}{2}\frac{\partial\mathrm{Q}^2}{\partial\lambda}\lambda'\mu'^2+\tfrac{1}{2}\frac{\partial\mathrm{R}^2}{\partial\lambda}\lambda'\nu'^2-\frac{\partial\mathrm{P}^2}{\partial\mu}\lambda'^2\mu'-\frac{\partial\mathrm{P}^2}{\partial\nu}\lambda'^2\nu'+\frac{\partial\phi}{\partial\lambda}\lambda'\Big)\\&+2b^2\Big(\tfrac{1}{2}\frac{\partial\mathrm{P}^2}{\partial\mu}\lambda'^2\mu'-\tfrac{1}{2}\frac{\partial\mathrm{Q}^2}{\partial\mu}\mu'^3+\tfrac{1}{2}\frac{\partial\mathrm{R}^2}{\partial\mu}\mu'\nu'^2-\frac{\partial\mathrm{Q}^2}{\partial\lambda}\lambda'\mu'^2-\frac{\partial\mathrm{Q}^2}{\partial\nu}\mu'^2\nu'+\frac{\partial\phi}{\partial\mu}\mu'\Big)\\&+2c^2\Big(\tfrac{1}{2}\frac{\partial\mathrm{P}^2}{\partial\nu}\lambda'^2\nu'+\tfrac{1}{2}\frac{\partial\mathrm{Q}^2}{\partial\nu}\mu'^2\nu'-\tfrac{1}{2}\frac{\partial\mathrm{R}^2}{\partial\nu}\nu'^3-\frac{\partial\mathrm{R}^2}{\partial\lambda}\lambda'\nu'^2-\frac{\partial\mathrm{R}^2}{\partial\mu}\mu'\nu'^2+\frac{\partial\phi}{\partial\nu}\nu'\Big) \quad (9)\end{aligned}$$

where in the last three lines the values of $\frac{\mathrm{D}\lambda'}{\mathrm{D}t}$ (or λ''), etc., have been inserted from the equations of motion (1).

Equation (9) has to be satisfied for all values of λ', μ', ν'; we therefore equate coefficients on both sides.

The coefficient of λ'^3 gives

$$\frac{\partial(a^2\mathrm{P}^2)}{\partial\lambda}-a^2\frac{\partial\mathrm{P}^2}{\partial\lambda}=0,$$

whence $$\frac{\partial a^2}{\partial\lambda}=0 \quad . \quad . \quad . \quad . \quad (10)$$

The coefficient of $\lambda'\mu'^2$ gives

$$\frac{\partial(b^2\mathrm{Q}^2)}{\partial\lambda}+a^2\frac{\partial\mathrm{Q}^2}{\partial\lambda}-2b^2\frac{\partial\mathrm{Q}^2}{\partial\lambda}=0,$$

therefore $$(a^2-b^2)\frac{\partial\mathrm{Q}^2}{\partial\lambda}=-\mathrm{Q}^2\frac{\partial b^2}{\partial\lambda}$$

$$=\mathrm{Q}^2\frac{\partial(a^2-b^2)}{\partial\lambda} \text{ by (10)}$$

Hence $$\frac{\partial}{\partial\lambda}\left(\frac{a^2-b^2}{\mathrm{Q}^2}\right)=0 \quad . \quad . \quad . \quad . \quad (11)$$

The coefficient of λ' gives $$\frac{\partial\kappa}{\partial\lambda}=2a^2\frac{\partial\phi}{\partial\lambda} \quad . \quad . \quad . \quad . \quad (12)$$

Collecting these and the symmetrical results, we obtain twelve equations, viz.

$$\left.\begin{array}{lll} \dfrac{\partial a^2}{\partial\lambda}=0 & \dfrac{\partial b^2}{\partial\mu}=0 & \dfrac{\partial c^2}{\partial\nu}=0 \\ \dfrac{\partial}{\partial\mu}\left(\dfrac{b^2-c^2}{\mathrm{R}^2}\right)=0 & \dfrac{\partial}{\partial\nu}\left(\dfrac{c^2-a^2}{\mathrm{P}^2}\right)=0 & \dfrac{\partial}{\partial\lambda}\left(\dfrac{a^2-b^2}{\mathrm{Q}^2}\right)=0 \\ \dfrac{\partial}{\partial\nu}\left(\dfrac{b^2-c^2}{\mathrm{Q}^2}\right)=0 & \dfrac{\partial}{\partial\lambda}\left(\dfrac{c^2-a^2}{\mathrm{R}^2}\right)=0 & \dfrac{\partial}{\partial\mu}\left(\dfrac{a^2-b^2}{\mathrm{P}^2}\right)=0 \\ \dfrac{\partial\phi}{\partial\lambda}=\dfrac{1}{2a^2}\dfrac{\partial\kappa}{\partial\lambda} & \dfrac{\partial\phi}{\partial\mu}=\dfrac{1}{2b^2}\dfrac{\partial\kappa}{\partial\mu} & \dfrac{\partial\phi}{\partial\nu}=\dfrac{1}{2c^2}\dfrac{\partial\kappa}{\partial\nu} \end{array}\right\} \quad (13)$$

The first nine equations of (13) enable us to find values of a, b, c appropriate to the curvilinear co-ordinates chosen, and so to determine the velocity-law consistent with a steady state of motion. It will be noticed that they involve neither the potential of the field of force nor the density.

The last three equations determine the density (involved in κ) when the field of force is given. If the force is due solely to the attractions of the stars themselves, an additional condition, viz. Poisson's equation, must be satisfied.

The Distribution of Velocities.

4. From the first nine equations of (13) an important general theorem can be derived.

The principal velocity-surfaces must be confocal quadrics. The general proof is rather difficult, and is given in an Appendix to this paper. We shall here give a proof for the most interesting case—that of axial symmetry,—which is much simpler.

When there is axial symmetry, two of the families of surfaces will consist of surfaces of revolution, and the third family, which cuts them orthogonally, must be the planes through the axis. We may take the co-ordinate ν corresponding to these planes to be the azimuthal angle ϕ; and all the quantities concerned must then, owing to the symmetry, be independent of ν.

Taking two of the equations of (13),

$$\frac{\partial}{\partial\lambda}\left(\frac{a^2-b^2}{\mathrm{Q}^2}\right)=0 \qquad \frac{\partial}{\partial\mu}\left(\frac{a^2-b^2}{\mathrm{P}^2}\right)=0,$$

these give on integration

$$a^2-b^2=\mathrm{Q}^2f_1(\mu)=\mathrm{P}^2f_2(\lambda) \quad . \quad . \quad . \quad (14)$$

where f_1 and f_2 are unknown functions.

Now, if (ϖ, z, ϕ) are cylindrical co-ordinates, the definitions of P and Q give

$$\mathrm{P}^2=\left(\frac{\partial\varpi}{\partial\lambda}\right)^2+\left(\frac{\partial z}{\partial\lambda}\right)^2$$

$$\mathrm{Q}^2=\left(\frac{\partial\varpi}{\partial\mu}\right)^2+\left(\frac{\partial z}{\partial\mu}\right)^2.$$

Introduce new parameters λ_0, μ_0 defined by

$$\lambda_0 = \int \frac{d\lambda}{\sqrt{f_2(\lambda)}} \qquad \mu_0 = \int \frac{d\mu}{\sqrt{f_1(\mu)}}.$$

Then $$\frac{\partial \varpi}{\partial \lambda_0} = \sqrt{f_2(\lambda)}\frac{\partial \varpi}{\partial \lambda}, \qquad \frac{\partial z}{\partial \lambda_0} = \sqrt{f_2(\lambda)}\frac{\partial z}{\partial \lambda}, \text{ etc.},$$

so that

$$\begin{aligned}\left(\frac{\partial \varpi}{\partial \lambda_0}\right)^2 + \left(\frac{\partial z}{\partial \lambda_0}\right) &= \mathrm{P}^2 f_2(\lambda) \\ &= \mathrm{Q}^2 f_1(\mu) \qquad \text{by (14)} \\ &= \left(\frac{\partial \varpi}{\partial \mu_0}\right)^2 + \left(\frac{\partial z}{\partial \mu_0}\right)^2\end{aligned}$$

Since λ_0 is a function of λ only, the surfaces $\lambda_0 = \text{const.}$ are identical with the surfaces $\lambda = \text{const.}$ Hence λ_0 can be used instead of λ as the parameter of the principal velocity-surfaces; so that (λ_0, μ_0, ν) could have been taken as our co-ordinates instead of (λ, μ, ν).* We shall suppose that λ_0, μ_0 have been used *ab initio*, and drop the suffixes as no longer necessary.

We have then

$$\left(\frac{\partial \varpi}{\partial \lambda}\right)^2 + \left(\frac{\partial z}{\partial \lambda}\right)^2 = \left(\frac{\partial \varpi}{\partial \mu}\right)^2 + \left(\frac{\partial z}{\partial \mu}\right)^2.$$

This is the condition that the orthogonal transformation ϖ, z to λ, μ may be *conformal*. Hence, by the properties of conformal transformations,

$$\varpi + \iota z = f(\lambda + \iota\mu), \quad \text{where} \quad \iota = \sqrt{-1}$$

and

$$\begin{aligned}\mathrm{P}^2 = \mathrm{Q}^2 &= \left(\frac{\partial \varpi}{\partial \lambda} + \iota\frac{\partial z}{\partial \lambda}\right)\left(\frac{\partial \varpi}{\partial \lambda} - \iota\frac{\partial z}{\partial \lambda}\right) \\ &= f'(\lambda + \iota\mu) f'(\lambda - \iota\mu).\end{aligned}$$

Again, by (14), since P^2 is now equal to Q^2,

$$\begin{aligned}a^2 - b^2 &= \mathrm{C}\mathrm{P}^2 \\ &= \mathrm{C} f'(\lambda + \iota\mu) f'(\lambda - \iota\mu),\end{aligned}$$

where C is a constant.

Operate on both sides by $\dfrac{\partial^2}{\partial \lambda \partial \mu}$. By (13) $\dfrac{\partial a^2}{\partial \lambda} = 0$, $\dfrac{\partial b^2}{\partial \mu} = 0$, so that the left side is annihilated. We obtain

$$\iota f'''(\lambda + \iota\mu) f'(\lambda - \iota\mu) - \iota f'''(\lambda - \iota\mu) f'(\lambda + \iota\mu) = 0.$$

Hence

$$\frac{f'''(\lambda + \iota\mu)}{f'(\lambda + \iota\mu)} = \frac{f'''(\lambda - \iota\mu)}{f'(\lambda - \iota\mu)}.$$

* In defining λ, μ, ν (p. 38) we only stated that they must be parameters of the surfaces; the particular parameter to be used was left optional.

The two sides involve different variables $\lambda+\iota\mu$ and $\lambda-\iota\mu$; since these may vary independently, each side must be constant and equal to p^2 say. Thus

$$f'''(\lambda+\iota\mu)=p^2f'(\lambda+\iota\mu).$$

This is a linear differential equation of which the complete solution is

$$\varpi+\iota z=f(\lambda+\iota\mu)=\mathrm{A}\cosh p(\lambda+\iota\mu+\epsilon)+\mathrm{B}. \quad . \quad (15)$$

where A, B, ϵ are constants of integration.

As is well known, the transformation (15) makes the curves $\lambda=$ const., $\mu=$ const. confocal conics; and the principal velocity-surfaces corresponding to λ and μ are formed by revolving these confocal conics about the z axis. The latter must evidently coincide with a principal axis of the conics, since otherwise a velocity-surface would intersect itself. This establishes the theorem.

In particular cases the quadrics may degenerate into spheres, cylinders, or planes.

An evident corollary is that if the system possesses *a plane of symmetry* the sections of the principal velocity-surfaces by that plane must be confocal conics. The plane of symmetry must be a principal velocity-surface, corresponding, say, to $\nu=$ const., and the argument applies unaltered so long as ν is not varied.

From these two results it appeared probable that the only possible forms of principal velocity-surfaces were confocal quadrics, and I was led to work out a general proof of this theorem, which is given in the Appendix (p. 54).

5. One of the families of principal velocity-surfaces consists of confocal ellipsoids. Take these to correspond to the λ co-ordinate. When we pass outwards along a curve orthogonal to these surfaces, $\mu=$ const., $\nu=$ const. Integrating

$$\frac{\partial a^2}{\partial\lambda}=0, \quad \frac{\partial}{\partial\lambda}\left(\frac{a^2-b^2}{\mathrm{Q}^2}\right)=0, \quad \text{and} \quad \frac{\partial}{\partial\lambda}\left(\frac{a^2-c^2}{\mathrm{R}^2}\right)=0, \text{ we have}$$

$$a^2=\mathrm{A}$$
$$b^2=\mathrm{A}+\mathrm{BQ}^2$$
$$c^2=\mathrm{A}+\mathrm{CR}^2$$

where A, B, C are constants for the curve.

Now Q and R must increase outwards without limit, since they measure the divergence of consecutive λ curves. Evidently therefore B and C must be taken positive; otherwise b^2 and c^2 would vanish at some point, and the corresponding velocities become infinite.

It follows that b^2 and c^2 are greater than a^2, so that the greatest axis of the velocity-ellipsoid is in the λ direction, *i.e.* more or less radial. The velocities in this—the star-streaming direction—are of the same magnitude in the outer parts of the system as near the centre, whereas the transverse velocities diminish according to the law $(\mathrm{A}+\mathrm{BQ}^2)^{-\frac{1}{2}}$ and $(\mathrm{A}+\mathrm{CR}^2)^{-\frac{1}{2}}$, which may be regarded as a

generalisation of the result $(A + Br^2)^{-\frac{1}{2}}$ found for the sphere in the previous paper.

6. Having shown that the principal velocity-surfaces are confocal quadrics, we now know the values of P, Q, R. The equations (13) are then easily solved. The general case is (*f*) below, but the particular cases when the quadrics degenerate into spheres, spheroids, etc., are especially useful. Only the results are given except for (*d*), which is worked out to show the method.

(*a*) *Rectangular co-ordinates* (x, y, z).

$$P = 1 \qquad Q = 1 \qquad R = 1.$$

We find a^2, b^2, c^2 are constants.

(*b*) *Cylindrical co-ordinates* (ϖ, z, ϕ).

$$P = 1 \qquad Q = 1 \qquad R = \varpi$$
$$a^2 = \text{const.} \qquad b^2 = \text{const.} \qquad c^2 = a^2(1 + \varpi^2).*$$

(*c*) *Spherical co-ordinates* (r, θ, ϕ).

$$P = 1 \qquad Q = r \qquad R = r \sin\theta$$
$$a^2 = \text{const.} \qquad b^2 = a^2(1 + r^2) \qquad c^2 = a^2(1 + r^2 + Cr^2 \sin^2\theta).$$

(*d*) *Prolate spheroidal co-ordinates* (ξ, η, γ).

$$x = \sinh\xi \sin\eta \cos\gamma \qquad y = \sinh\xi \sin\eta \sin\gamma \qquad z = \cosh\xi \cos\eta$$
$$P^2 = Q^2 = \cosh^2\xi - \cos^2\eta \qquad R^2 = \sinh^2\xi \sin^2\eta.$$

From (13) we have, since $P^2 = Q^2$,

$$\frac{\partial}{\partial\xi}\left(\frac{a^2 - b^2}{P^2}\right) = 0 \qquad \frac{\partial}{\partial\eta}\left(\frac{a^2 - b^2}{P^2}\right) = 0$$
$$\frac{\partial}{\partial\gamma}\left(\frac{a^2 - b^2}{P^2}\right) = -\frac{\partial}{\partial\gamma}\left(\frac{b^2 - c^2}{P^2}\right) - \frac{\partial}{\partial\gamma}\left(\frac{c^2 - a^2}{P^2}\right) = 0.$$

Hence, integrating

$$a^2 - b^2 = -BP^2 = -B(\cosh^2\xi - \cos^2\eta)$$

where $-B$ is a constant of integration.

Since $\partial a^2/\partial\xi = 0$ $\partial b^2/\partial\eta = 0$,

this gives $a^2 = A + B\cos^2\eta$ $b^2 = A + B\cosh^2\xi$.

By similar use of the remaining equations it can be shown that

$$c^2 = A + B\sinh^2\xi + B\cos^2\eta + C\sinh^2\xi \sin^2\eta.$$

(*e*) *Oblate co-ordinates* (ξ, η, γ).

$$x = \cosh\xi \cos\eta \cos\gamma \qquad y = \cosh\xi \cos\eta \sin\gamma \qquad z = \sinh\xi \sin\eta$$
$$P^2 = Q^2 = \cosh^2\xi - \cos^2\eta \qquad R^2 = \cosh^2\xi \cos^2\eta$$
$$a^2 = A + B\cos^2\eta \qquad b^2 = A + B\cosh^2\xi$$
$$c^2 = A + B\cosh^2\xi + B\cos^2\eta + C\cosh^2\xi \cos^2\eta.$$

(*f*) *General ellipsoidal co-ordinates* (λ, μ, ν).

* We omit such constants as may be absorbed by changing the unit of measurement.

The squares of the semiaxes of the confocals through (λ, μ, ν) being

$$\lambda^2, \lambda^2-\beta^2, \lambda^2-\gamma^2; \qquad \mu^2, \mu^2-\beta^2, \mu^2-\gamma^2; \qquad \nu^2, \nu^2-\beta^2, \nu^2-\gamma^2$$

$$P^2=\frac{(\lambda^2-\mu^2)(\lambda^2-\nu^2)}{(\lambda^2-\beta^2)(\lambda^2-\gamma^2)} \quad Q^2=\frac{(\mu^2-\nu^2)(\lambda^2-\mu^2)}{(\mu^2-\beta^2)(\gamma^2-\mu^2)} \quad R^2=\frac{(\lambda^2-\nu^2)(\mu^2-\nu^2)}{(\beta^2-\nu^2)(\gamma^2-\nu^2)}$$

$$a^2=A+B\mu^2+B\nu^2+C\mu^2\nu^2 \qquad b^2=A+B\nu^2+B\lambda^2+C\nu^2\lambda^2$$

$$c^2=A+B\lambda^2+B\mu^2+C\lambda^2\mu^2.$$

The Distribution of Density.

7. When the field of force is given, the last three equations of (13) determine the density. But the force must satisfy a certain condition in order that the ellipsoidal law may be possible. We have by the last three equations of (13)

$$a^2\frac{\partial\phi}{\partial\lambda}d\lambda+b^2\frac{\partial\phi}{\partial\mu}d\mu+c^2\frac{\partial\phi}{\partial\nu}d\nu=\frac{1}{2}\frac{\partial\kappa}{\partial\lambda}d\lambda+\frac{1}{2}\frac{\partial\kappa}{\partial\mu}d\mu+\frac{1}{2}\frac{\partial\kappa}{\partial\nu}d\nu=\tfrac{1}{2}d\kappa \quad (16)$$

so that ϕ must be of a form which makes the left side of (16) a perfect differential. This requires that

$$\frac{\partial}{\partial\mu}\left(a^2\frac{\partial\phi}{\partial\lambda}\right)=\frac{\partial}{\partial\lambda}\left(b^2\frac{\partial\phi}{\partial\mu}\right), \text{ etc.}$$

Hence
$$\frac{\partial a^2}{\partial\mu}\frac{\partial\phi}{\partial\lambda}-\frac{\partial b^2}{\partial\lambda}\frac{\partial\phi}{\partial\mu}+(a^2-b^2)\frac{\partial^2\phi}{\partial\lambda\partial\mu}=0.$$

Or, since
$$\frac{\partial a^2}{\partial\lambda}=0, \frac{\partial b^2}{\partial\mu}=0,$$

$$\phi\frac{\partial^2}{\partial\lambda\partial\mu}(a^2-b^2)+\frac{\partial(a^2-b^2)}{\partial\mu}\frac{\partial\phi}{\partial\lambda}+\frac{\partial(a^2-b^2)}{\partial\lambda}\frac{\partial\phi}{\partial\mu}+(a^2-b^2)\frac{\partial^2\phi}{\partial\lambda\partial\mu}=0.$$

Therefore
$$\frac{\partial^2}{\partial\lambda\partial\mu}\{(a^2-b^2)\phi\}=0 \quad . \quad . \quad . \quad (17)$$

Consider the case of axial symmetry, so that the quantities are independent of ν. Integrating (17)

$$\phi=\frac{f(\lambda)+g(\mu)}{a^2-b^2} \quad . \quad . \quad . \quad (18)$$

where f and g are arbitrary functions.

The value of κ is then

$$\kappa=2\cdot\frac{a^2f(\lambda)+b^2g(\mu)}{a^2-b^2} \quad . \quad . \quad . \quad (19)$$

For, writing this in the form

$$\kappa=2a^2\frac{f(\lambda)+g(\mu)}{a^2-b^2}-2g(\mu),$$

we see that $\frac{\partial\kappa}{\partial\lambda}=2a^2\frac{\partial\phi}{\partial\lambda}$; and similarly $\frac{\partial\kappa}{\partial\mu}=2b^2\frac{\partial\phi}{\partial\mu}$ is satisfied.

σ is then given by equation (8*a*)

$$\kappa = \log(abc\sigma) \quad . \quad . \quad . \quad . \quad (20)$$

When the system moves under its own attraction only, Poisson's equation $\nabla^2\phi = -4\pi\sigma$ must also be satisfied. Substituting for ϕ and σ we have in this case

$$\nabla^2 \left\{ \frac{f(\lambda)+g(\mu)}{a^2-b^2} \right\} = -\frac{4\pi}{abc} \exp. \left\{ 2\frac{a^2 f(\lambda)+b^2 g(\mu)}{a^2-b^2} \right\}.$$

It is easy to prove that this cannot be exactly satisfied for spheroidal co-ordinates. In fact, *the only possible solution is the case of spherical symmetry* (excluding a very artificial case of two-dimensional motion with circular symmetry). It may conveniently be added here that a superimposed rotation (considered in § 9) does not lead to any further solution.

If, on the other hand, the motion takes place under the attraction of a controlling distribution whose potential is of the form (18), there are no further conditions to be satisfied; solutions for spheroidal and ellipsoidal velocity-surfaces can be obtained. It seems best to assume that the controlling system has spherical symmetry, for otherwise there remains the awkward question how its own equilibrium is maintained; but the controlled system can take an oblate or prolate form.

8. As a first example, take the case of spherical co-ordinates, § 6 (*c*). Equations (18) and (19) become

$$\phi = -\frac{f(r)+g(\theta)}{r^2} \qquad \kappa = 2a^2 \cdot \frac{f(r)+(1+r^2)g(\theta)}{r^2}.$$

Unless $g(\theta)$ is a constant, ϕ, κ, and σ become infinite and (worse still) physically discontinuous at the centre. Thus the only practical solution is when κ is a function of r only, and therefore ϕ is a function of r only.

If the system moves under its own attraction, it follows that σ is a function of r only, and we have the case of complete spherical symmetry treated in the Second Paper (the formulæ of that paper follow immediately). But if the system moves under more general forces

$$abc\sigma = e^{\kappa} = \text{a function of } r \text{ only},$$

and since

$$c^2 = 1 + r^2 + Cr^2 \sin^2\theta, \ \sigma \text{ will vary with } \theta \text{ unless C is zero.}$$

Thus even when the principal velocity-surfaces are spheres we can have a prolate or oblate distribution of the stars, provided that the system is controlled by the attraction of a globular distribution of matter. Theoretically the degree of oblateness might be as great as we please, but the observational results (p. 38) require that $b^2 \simeq c^2$ in the neighbourhood of the Sun ($r = \sqrt{3}$), so that C must be taken very small, and the distribu-

tion of density will then differ only slightly from spherical symmetry.*

Before passing on to consider spheroidal velocity-surfaces, we shall examine whether it is possible to obtain greater oblateness by imposing a rotation of the whole system about the galactic axis.

9. *Effect of a Superposed Rotation.*—Assuming axial symmetry, let ν be the azimuthal angle ϕ, and let ω be the angular velocity of rotation, which will in general be a function of λ and μ. We must then replace ν' by $(\nu' - \omega)$ in Schwarzschild's ellipsoidal formula, so that (2) becomes

$$\mathrm{N} = \pi^{-\frac{3}{2}} abc\sigma \mathrm{P}^2\mathrm{Q}^2\mathrm{R}^2 d\lambda d\mu d\nu d\lambda' d\mu' d\nu' \exp \{ -a^2\mathrm{P}^2\lambda'^2 - b^2\mathrm{Q}^2\mu'^2 - c^2\mathrm{R}^2(\nu' - \omega)^2 \}$$

and (8) becomes

$$\frac{\mathrm{D}}{\mathrm{D}t}\log(abc\sigma) = \frac{\mathrm{D}}{\mathrm{D}t}\{a^2\mathrm{P}^2\lambda'^2 + b^2\mathrm{Q}^2\mu'^2 + c^2\mathrm{R}^2\nu'^2 - 2c^2\mathrm{R}^2\nu'\omega + c^2\mathrm{R}^2\omega^2\}.$$

Consider the additional term $\mathrm{D}(2c^2\mathrm{R}^2\nu'\omega)/\mathrm{D}t$. By Lagrange's equations $\mathrm{D}(\mathrm{R}^2\nu')/\mathrm{D}t = 0$ for axial symmetry. Hence

$$\frac{\mathrm{D}}{\mathrm{D}t}(2c^2\mathrm{R}^2\nu'\omega) = 2\mathrm{R}^2\nu'\left(\lambda'\frac{\partial}{\partial\lambda} + \mu'\frac{\partial}{\partial\mu}\right)(c^2\omega).$$

This addition, being of the second degree in the velocities, will not combine with any of the other terms in (9) and must vanish independently of them. Thus $c^2\omega$ must be a constant, equal to Ω, say. Thus

$$\omega = \Omega/c^2 \qquad . \quad . \quad . \quad . \quad (21)$$

The other additional term $\exp(c^2\mathrm{R}^2\omega^2) = \exp(\mathrm{R}^2\Omega^2/c^2)$ does not contain the velocities and can be incorporated in κ.

Thus, without altering any of the formulæ (13), a rotation varying as c^{-2} can be superimposed, but in place of (20) we have

$$\log(abc\sigma) = \kappa + \mathrm{R}^2\Omega^2/c^2.$$

10. Applying the last result to the case of spherical co-ordinates considered in § 8,

$$\log(abc\sigma) = \kappa + \frac{\Omega^2 r^2 \sin^2\theta}{a^2(1 + r^2 + \mathrm{C}r^2\sin^2\theta)} \qquad . \quad . \quad (22)$$

and κ has been shown to be a function of r only.

We have seen that C must be small if not zero; we shall therefore take it to be zero.

Comparing the density σ_θ with the density σ_0 at the same distance along the galactic axis, equation (22) gives

$$\log\frac{\sigma_\theta}{\sigma_0} = \frac{\Omega^2}{a^2}\cdot\frac{r^2}{1 + r^2}\sin^2\theta\,.$$

* A large value of C is also objectionable on theoretical grounds (see p. 51).

This increases as θ increases from 0° to 90°, so that the distribution is oblate; and the oblateness may be made as large as we please by taking Ω sufficiently great.

To obtain an estimate of the necessary amount of rotation, we use the result that the number of faint stars in the galactic plane is about four times as great as along the galactic axis. Thus for large values of r, $\sigma_{90}/\sigma_0 \doteq 4$. Hence

$$\Omega = a\sqrt{\log 4}$$

and

$$\omega = \frac{\sqrt{\log 4}}{a(1+r^2)}.$$

As in the Second Paper, the unit of r is $1/\sqrt{3}$ times the Sun's distance from the centre, and $a = 1$ when the unit of velocity is about 50 km. per sec. The stars near the Sun are therefore in the mean moving round the galactic axis with a linear velocity.

$$= \sqrt{3}\,\omega = \frac{\sqrt{3\log 4}}{4} \times 50 \text{ km. per sec.}$$

$$= 25 \text{ km. per sec.}$$

Unless the Sun is a great distance from the centre of the system, a rotation so large as this could scarcely have escaped observation. One circumstance might, however, tend to conceal it; the linear velocity of rotation, varying with $r/(1+r^2)$, reaches a maximum of less than 30 km. per sec. at $r = 1$. The law of angular velocity is, in fact, such as to give comparatively little differential motion within, say, unit distance of the Sun.

Prolate Velocity-Surfaces.

11. We shall now consider the case of prolate co-ordinates (§ 6, *d*)—the most important case, because it gives a model of the stellar system agreeing satisfactorily with observation in most respects. It may seem paradoxical that in order to obtain an *oblate* distribution of stars we should take *prolate* velocity-surfaces; but the reason is clear. The normal to a prolate spheroid is a compromise between the direction towards the centre and the direction parallel to the galactic plane (the axis of symmetry being towards the galactic poles). We thus secure that the preferential motion is more nearly parallel to the galactic plane than in the case of spherical symmetry, and this tends to keep the stars closer to the plane. With oblate velocity-surfaces, the directions of preferential motion would be diverted away from the plane, and the stars would tend to spread out at right angles to it.

We shall assume, as before, that the stars move under a field of force due to a globular controlling distribution, and further, that

this controlling distribution is of uniform density.* If the density of the controlling matter is $\frac{3E}{2\pi}$, the potential is (neglecting a constant)

$$\phi = -E(1+r^2).$$

In prolate co-ordinates this gives

$$\begin{aligned}\phi &= -E(\cosh^2\xi+\cos^2\eta)\\ &= EB\frac{(\cosh^4\xi-\cos^4\eta)}{(A+B\cos^2\eta)-(A+B\cosh^2\xi)}.\end{aligned}$$

This is of the form (18), hence by (19)

$$\begin{aligned}\kappa &= 2EB\frac{(A+B\cos^2\eta)\cosh^4\xi-(A+B\cosh^2\xi)\cos^4\eta}{(A+B\cos^2\eta)-(A+B\cosh^2\xi)}\\ &= -2E(A\cosh^2\xi+A\cos^2\eta+B\cosh^2\xi\cos^2\eta) \qquad (23)\\ &= \log(abc\sigma)\end{aligned}$$

Confining ourselves to distances from the centre greater than unity,

$\eta = 0°$ for the galactic pole $\eta = 90°$ for the galactic plane

$\cosh\xi = r$,, ,, $\sinh\xi = r$,, ,,

Hence by § 6, *d*, and (23), for the galactic pole :

$$abc = \sqrt{(A+B)(A+Br^2)(A+Br^2)} \qquad \kappa = -2E(A+(A+B)r^2) \quad (24)$$

for the galactic plane :

$$abc = \sqrt{A(A+B+Br^2)(A+Br^2+Cr^2)} \qquad \kappa = -2E(A+Ar^2) \quad (25)$$

The same expressions are found for abc and κ when the distance from the centre is less than unity.

The ratio of the densities at the same distance towards the galactic pole and in the galactic plane is thus (by (20), (24), and (25))

$$\frac{\sigma_0}{\sigma_{90}} = \sqrt{\frac{A(A+B+Br^2)(A+Br^2+Cr^2)}{(A+B)(A+Br^2)^2}}\cdot e^{-2EBr^2}.$$

Evidently when r is large the factor e^{-2EBr^2} predominates so that the ratio tends to zero. Thus in the outer parts the system is strongly oblate.

When r is small, we have, by expanding,

$$\frac{\sigma_0}{\sigma_{90}} = 1+\tfrac{1}{2}\left(\frac{B}{A+B}+\frac{C-B}{A}-4EB\right)r^2+\ \ldots\ .$$

* The assumption of uniform density is necessary in order to obtain an exact solution. Admittedly it could not occur in nature. But from my point of view the artificial character of the restriction is not fatal. If the density is not uniform, the system will not be steady, and the ellipsoidal law will probably not be conserved ; but the simpler system may be expected to bear some general resemblance to the more natural system.

Unless C is large, the system will be oblate right up to the centre. We have, of course, no observational knowledge as to whether the central parts are oblate or not.

There is one difficulty in making these formulæ fit the facts of observation which perhaps tells in favour of spherical instead of prolate velocity-surfaces. For stars near the Sun the observations give $b^2 \simeq c^2 \simeq 4a^2$. If r_0 is the distance of the Sun from the centre (the unit of distance being half the focal line of the confocals), this gives

$$A + B + Br_0^2 \simeq A + Br_0^2 + Cr_0^2 \simeq 4A.$$

It is improbable that C is other than a very small quantity; for a large value of C would mean that c^2 becomes greater than b^2 in the outer parts of the system, *i.e.* the motion would tend to take place mainly in planes through the galactic axis. I can see no way in which such an unnatural arrangement could have arisen. But if C is small it follows that r_0 must be rather large, and the confocal surfaces at and beyond the Sun are beginning to approximate to spheres. By giving a large numerical value to r_0 (diminishing the unit of distance) we are restricting the space in which the velocity surfaces are sensibly elliptic and tending to revert to the case of spherical velocity-surfaces. Perhaps, however, we are laying too much stress on the approximate equality of b^2 and c^2; the result comes from averaging stellar velocities distributed over a large volume of space, and may not strictly represent the velocity-ellipsoid at a *point*. Moreover, it may be significant that for type Ao, for which the oblateness is especially pronounced, the equality does not hold, and both C and r_0 may be taken small.

In all my formulæ the falling off of density outwards appears, at least at first sight, to be too rapid to be reconciled with observation. By (25) the density in the galactic plane falls off at least as rapidly as $\exp(-2EAr^2)$; actually there is another factor which makes the diminution still more rapid. This can be interpreted to mean that the stars thin out in the ratio e^{-1} when we reach a distance r such that the velocity in a circular orbit of radius r in the controlling medium is equal to $\sqrt{\pi}$ times the average velocity of the stars resolved in the star-streaming direction. In the Second Paper I took this average velocity to be 28 km. per sec., which multiplied by $\sqrt{\pi}$ gives 50 km. per sec. The latter was taken as the unit of velocity, and with this unit $A = a^2 = 1$. I do not think we can take the distance at which the falling off of density e^{-1} occurs in the galactic plane to be less than 800 parsecs. These numbers give for the density of the controlling medium 110 times the Sun's mass in a sphere of five parsecs radius. This is perilously near the minimum estimate of density of the lucid stars, viz. 10 times the Sun's mass in the same sphere.

To meet this difficulty it may be pointed out that the heterogeneity of the stellar system affects this problem considerably. The average speed of 28 kilometres per second depends on the more luminous stars, and it is possible that these may thin out

rather rapidly. There exists also a class of intrinsically faint stars with much higher speeds, which, according to the formulæ, will be more extended in distribution. This consideration permits us to increase the calculated density of the controlling medium perhaps 4 or 5 times. Again, we have taken the controlling distribution to be of uniform density; if, however, it decreases outwards, the stellar density will not diminish so fast.

The reason why our formulæ limit the number of distant stars is that a certain proportion of them must have orbits passing near the centre, and these will acquire great velocities in their fall. Thus the number of stars allowable in the outer parts depends on the proportion of great velocities near the centre. On almost any theory relating to an approximately steady state a correspondence of this kind would occur; and it may be noted that the time required to establish a rough equilibrium relation between the inner and outer parts is probably not greater than 100 million years. If we could assert definitely that there are too many distant stars to correspond to the observed motions at the centre, it would, I think, tend to show that the system is undergoing a very rapid collapse or dispersal.

However, on the whole, it seems possible to represent the observed results with about as good a quantitative agreement as could be expected from a theory, which necessarily omits some of the complexities of the actual universe; in regard to the two points mentioned, I feel that the agreement is a little strained, and the margin of safety is smaller than is desirable.

12. *Prolate Velocity-Surfaces with Rotation.*—The two methods of obtaining an oblate distribution of density, discussed in §§ 10 and 11 respectively, can be combined, *i.e.* we can take prolate velocity-surfaces and superimpose rotation. Naturally it is easier to account for the large oblateness by the two causes than by either of them singly; moreover, it seems natural to expect that in an actual system both causes would act. There is also some relief to the rapid falling off of density in the galactic plane when rotation is imposed, as can be seen from the formulæ.

For prolate surfaces the angular velocity is by (21)

$$\omega = \frac{\Omega}{A + B\sinh^2\xi + B\cos^2\eta + C\sinh^2\xi\sin^2\eta},$$

which becomes for the galactic plane

$$\omega = \frac{\Omega}{A + (B + C)r^2},$$

and the rotation increases the density in the galactic plane by the factor

$$\exp \frac{r^2\Omega^2}{A + (B + C)r^2},$$

the density towards the poles being unaltered.

The complete expressions for the density are

$$\left.\begin{aligned}\sigma_0 &= \frac{\exp\{-2\mathrm{E}(\mathrm{A}+\mathrm{B})r^2\}}{\sqrt{\{(\mathrm{A}+\mathrm{B})(\mathrm{A}+\mathrm{B}r^2)^2\}}}\\ \sigma_{90} &= \frac{\exp\{-2\mathrm{E}\mathrm{A}r^2 + r^2\Omega^2/(\mathrm{A}+\mathrm{B}r^2+\mathrm{C}r^2)\}}{\sqrt{\{\mathrm{A}(\mathrm{A}+\mathrm{B}+\mathrm{B}r^2)(\mathrm{A}+\mathrm{B}r^2+\mathrm{C}r^2)\}}}\end{aligned}\right\} \quad . \quad (26)$$

It may be noticed that this last expression by a suitable choice of constants may be made to increase at first, and finally decrease as r becomes larger. We could thus have a ring formed—the Milky Way. But I doubt whether such a theory of the galaxy would work out satisfactorily in detail.

13. *Systems in an Unsteady State.*—The observed velocities are found to obey approximately the ellipsoidal law, and the present paper has dealt with systems which satisfy the law rigorously. If we attach importance to the law, it is reasonable to suppose that, however the system may change in the course of time, this approximate agreement will tend to continue; otherwise we should have to attribute the observed agreement to a coincidence, depending on the particular stage of evolution at which we happen to be living. We may therefore consider briefly a system in a changing state, but changing so as still to conform to the ellipsoidal law.

In that case σ, a, b, c, κ, ϕ will be functions of the time. Further, since the principal velocity-surfaces may be changing, there will be product terms in the exponent in equation (2) such as $-2f\mathrm{R}\mu'\nu'$, where f is zero now but $\frac{\partial f}{\partial t}$ is not zero. We shall also allow in the exponent terms linear in the velocities, indicating that one part of the system is moving as a whole relatively to another part (*cf.* the superposing of a rotation, § 9); and the coefficients of these terms may also vary with the time.

This generalisation involves a great many extra terms in (9); but it will be seen on making the calculation that no addition is made to the terms of the third degree in λ', μ', ν'. Accordingly the first nine equations of (13) are true even for an unsteady system, and all the results relating to the velocity-distribution, §§ 4–6, remain valid.

Most of the new terms are of the second degree, and merely lead to additional conditions to be satisfied by the time-derivatives of the coefficients, etc. But there are also linear terms, so that the last three equations of (13) are modified. Accordingly the results relating to the density-distribution, §§ 7–12, will not hold in an unsteady system.

I have not followed out any further the problem of a system in a changing state; but I do not think the solution, at least for the case of axial symmetry, would be difficult. The general validity of the formulæ for the velocity-distribution makes an extraordinary simplification. The principal velocity-surfaces are confocals of revolution and can change in one way only, by varying

the distance apart of the foci. The lengths of the axes of the velocity-ellipsoids at all points are determined by the three constants A, B, C, which must now be supposed to vary with the time. Thus the most general specification of the velocity-ellipsoids is at once reduced to four parameters varying with the time.

[*Added Nov.* 29.—I have now examined this problem. The result is that no exact solution exists. We infer that, in the course of collapsing from an oblate to a spherical form, the system must necessarily deviate from the ellipsoidal law.]

Appendix.

General proof of the theorem that the principal velocity-surfaces must be confocal quadrics.

In the following argument the letters α, β, γ (with or without suffixes) will be used to denote arbitrary functions of λ, μ, ν respectively, and the letters a, b, c to denote functions of (μ, ν), (ν, λ), (λ, μ) respectively. The unsuffixed a, b, c already in use agree with this convention. It is very necessary to bear these meanings in mind in following the steps of the proof, as continual use is being made of these properties, usually without explicit mention. The letters A, B, C are used for arbitrary constants.

Integrating the middle six equations of (13), we have

$$\left.\begin{aligned} a^2 - b^2 &= \mathrm{P}^2 b_1 = \mathrm{Q}^2 a_2 \\ b^2 - c^2 &= \mathrm{Q}^2 c_1 = \mathrm{R}^2 b_2 \\ c^2 - a^2 &= \mathrm{R}^2 a_1 = \mathrm{P}^2 c_2 \end{aligned}\right\} \quad . \quad . \quad . \quad (27)$$

from which

$$a_1 b_1 c_1 = a_2 b_2 c_2 \quad . \quad . \quad . \quad . \quad (28)$$

Now if a_1 contains a factor involving μ and ν inseparably (*i.e.* not of the form $\beta\gamma$), the only other term in (28) in which such a factor can occur is a_2, so that a_2 must be of the form $a_1\beta\gamma$. We can include all possible cases by writing

Similarly

$$\left.\begin{aligned} a_1 &= \gamma_1 a_3 & a_2 &= \beta_2 a_3 \\ b_1 &= \alpha_1 b_3 & b_2 &= \gamma_2 b_3 \\ c_1 &= \beta_1 c_3 & c_2 &= \alpha_2 c_3 \end{aligned}\right\} \quad . \quad . \quad . \quad (29)$$

hence by (28)

$$\alpha_1\beta_1\gamma_1 = \alpha_2\beta_2\gamma_2.$$

By a similar argument we see that a_1 can only differ from a_2 by a constant multiplier, so that

$$a_1 = \mathrm{A}a_2 \qquad \beta_1 = \mathrm{B}\beta_2 \qquad \gamma_1 = \mathrm{C}\gamma_2 \quad \text{and} \quad \mathrm{ABC} = 1.$$

By writing

$$\left.\begin{aligned} \gamma_1 &= \gamma_3 & \beta_1 &= \mathrm{B}\beta_3 & a_1 &= a_3/\mathrm{C} \\ a_3 &= a_0 & b_3 &= \mathrm{C}b_0 & c_3 &= c_0/\mathrm{B} \end{aligned}\right\} \quad . \quad . \quad (30)$$

the constants are absorbed and we get from (27)

$$\left.\begin{aligned} a^2 - b^2 &= \mathrm{P}^2 a_3 b_0 = \mathrm{Q}^2 \beta_3 a_0 \\ b^2 - c^2 &= \mathrm{Q}^2 \beta_3 c_0 = \mathrm{R}^2 \gamma_3 b_0 \\ c^2 - a^2 &= \mathrm{R}^2 \gamma_3 a_0 = \mathrm{P}^2 \alpha_3 c_0 \end{aligned}\right\} \quad . \quad . \quad . \quad (31)$$

from which

$$\frac{P^2\alpha_3}{a_0}=\frac{Q^2\beta_3}{b_0}=\frac{R^2\gamma_3}{c_0}=\frac{a^2-b^2}{a_0b_0}=\frac{b^2-c^2}{b_0c_0}=\frac{c^2-a^2}{c_0a_0} \quad . \quad (32)$$

From the last three members of (32)

$$a_0b_0+b_0c_0+c_0a_0=0,$$

hence

$$a_0^{-1}+b_0^{-1}+c_0^{-1}=0 \quad . \quad . \quad . \quad (33)$$

Differentiate with respect to λ; we obtain

$$\frac{\partial c_0^{-1}}{\partial\lambda}=-\frac{\partial b_0^{-1}}{\partial\lambda}.$$

And since one side does not contain ν, and the other does not contain μ, each side must be a function of λ only. Call this function $-\frac{\partial\alpha}{\partial\lambda}$.

$$\frac{\partial c_0^{-1}}{\partial\lambda}=-\frac{\partial\alpha}{\partial\lambda} \qquad \frac{\partial b_0^{-1}}{\partial\lambda}=\frac{\partial\alpha}{\partial\lambda},$$

whence integrating (remembering that $\partial c_0^{-1}/\partial\nu=0$, $\partial b_0^{-1}/\partial\mu=0$)

$$c_0^{-1}=\beta-\alpha \qquad b_0^{-1}=\alpha-\gamma,$$

and from (33) $$a_0^{-1}=\gamma-\beta.$$

Hitherto λ, μ, ν have not been exactly specified; any parameters of the principal velocity-surfaces might be used. As in § 4 we may use for our co-ordinates any functions of the original λ, μ, ν without altering the frame of reference. We shall choose α^{-1}, β^{-1}, γ^{-1} as the new λ, μ, ν. Of course in doing so we alter P, Q, R. But the new P differs only from the old P by a factor which, being a function of λ only, can be absorbed in α_3.

Hence $$a_0=\frac{\mu\nu}{\mu-\nu} \qquad b_0=\frac{\nu\lambda}{\nu-\lambda} \qquad c_0=\frac{\lambda\mu}{\lambda-\mu} \quad . \quad . \quad (34)$$

and (32) becomes

$$\frac{P^2\alpha_3(\mu-\nu)}{\mu\nu}=\frac{Q^2\beta_3(\nu-\lambda)}{\nu\lambda}=\frac{R^2\gamma_3(\lambda-\mu)}{\lambda\mu}=\frac{(a^2-b^2)(\mu-\nu)(\nu-\lambda)}{\lambda\mu\nu^2}=\ldots.$$

Multiplying by $$\frac{\lambda\mu\nu}{(\lambda-\mu)(\mu-\nu)(\lambda-\nu)}$$

$$\frac{P^2\alpha_0}{(\lambda-\mu)(\lambda-\nu)}=\frac{Q^2\beta_0}{(\mu-\lambda)(\mu-\nu)}=\frac{R^2\gamma_0}{(\nu-\lambda)(\nu-\mu)}=\frac{a^2-b^2}{\nu(\mu-\lambda)}$$
$$=\frac{b^2-c^2}{\lambda(\nu-\mu)}=\frac{c^2-a^2}{\mu(\lambda-\nu)}=S \quad (35)$$

where α_0 is written for $\alpha_3\lambda$. S may be any function of λ, μ, ν.

We now introduce six new differential equations, which must be satisfied by P, Q, R. These are known as the Lamé relations,

and embody the geometrical conditions satisfied by a triply orthogonal system of surfaces.* The Lamé relations are

$$\frac{\partial^2 P}{\partial\mu\partial\nu} - \frac{1}{Q}\frac{\partial Q}{\partial\nu}\frac{\partial P}{\partial\mu} - \frac{1}{R}\frac{\partial R}{\partial\mu}\frac{\partial P}{\partial\nu} = 0 \quad . \quad . \quad (36)$$

$$\frac{\partial}{\partial\lambda}\left(\frac{1}{P}\frac{\partial Q}{\partial\lambda}\right) + \frac{\partial}{\partial\mu}\left(\frac{1}{Q}\frac{\partial P}{\partial\mu}\right) + \frac{1}{R^2}\frac{\partial P}{\partial\nu}\frac{\partial Q}{\partial\nu} = 0 \quad . \quad . \quad (37)$$

and the corresponding cyclic results.

We can write (36)

$$\frac{\partial^2}{\partial\mu\partial\nu}\log P = \frac{\partial\log Q}{\partial\nu}\frac{\partial\log P}{\partial\mu} + \frac{\partial\log R}{\partial\mu}\frac{\partial\log P}{\partial\nu} - \frac{\partial\log P}{\partial\nu}\frac{\partial\log P}{\partial\mu}.$$

Now from (35)

$$\left.\begin{aligned} 2\log P &= \log(\lambda-\mu) + \log(\lambda-\nu) + \log S - \log\alpha_0 \\ 2\log Q &= \log(\mu-\lambda) + \log(\mu-\nu) + \log S - \log\beta_0 \\ 2\log R &= \log(\nu-\lambda) + \log(\nu-\mu) + \log S - \log\gamma_0 \end{aligned}\right\} \quad . \quad (38)$$

Thus (36) becomes

$$\begin{aligned} 2\frac{\partial^2}{\partial\mu\partial\nu}\log S = {} & \left(\frac{\partial\log S}{\partial\nu} + \frac{1}{\nu-\mu}\right)\left(\frac{\partial\log S}{\partial\mu} + \frac{1}{\mu-\lambda}\right) \\ & + \left(\frac{\partial\log S}{\partial\mu} + \frac{1}{\mu-\nu}\right)\left(\frac{\partial\log S}{\partial\nu} + \frac{1}{\nu-\lambda}\right) \\ & - \left(\frac{\partial\log S}{\partial\nu} + \frac{1}{\nu-\lambda}\right)\left(\frac{\partial\log S}{\partial\mu} + \frac{1}{\mu-\lambda}\right) \\ = {} & \frac{\partial\log S}{\partial\mu}\frac{\partial\log S}{\partial\nu} + \frac{1}{\nu-\mu}\left(\frac{\partial\log S}{\partial\mu} - \frac{\partial\log S}{\partial\nu}\right) \quad . \quad (39) \end{aligned}$$

Whence

$$2\frac{\partial^2 S}{\partial\mu\partial\nu} = \frac{3}{S}\frac{\partial S}{\partial\mu}\frac{\partial S}{\partial\nu} + \frac{1}{\nu-\mu}\left(\frac{\partial S}{\partial\mu} - \frac{\partial S}{\partial\nu}\right) \quad . \quad . \quad (40)$$

Again, from (35)

$$\frac{b^2-c^2}{\lambda} = S(\nu-\mu),$$

operate by $\frac{\partial^2}{\partial\mu\partial\nu}$, the left side vanishes, and we have

$$\frac{\partial^2 S}{\partial\mu\partial\nu} + \frac{1}{\nu-\mu}\left(\frac{\partial S}{\partial\mu} - \frac{\partial S}{\partial\nu}\right) = 0 \quad . \quad . \quad . \quad (41)$$

From (40) and (41)

$$\frac{\partial^2 S}{\partial\mu\partial\nu} = \frac{1}{S}\frac{\partial S}{\partial\mu}\frac{\partial S}{\partial\nu},$$

from which (dividing through by $\frac{\partial S}{\partial\mu}$ and integrating with respect to ν)

$$\log\frac{\partial S}{\partial\mu} = \log S + c_0,$$

hence

$$\frac{\partial S}{\partial\mu} = c_1 S.$$

* See, for example, Forsyth, *Differential Geometry*, p. 418.

From the symmetrical equation we could obtain similarly

$$\frac{\partial S}{\partial \mu} = a_1 S,$$

and these two results can only be reconciled by writing

$$\frac{\partial S}{\partial \mu} = \beta_1 S \qquad . \quad . \quad . \quad . \quad (42)$$

and there will be similar results for λ and ν.

The equations

$$\frac{1}{S}\frac{\partial S}{\partial \lambda} = a_1 \qquad \frac{1}{S}\frac{\partial S}{\partial \mu} = \beta_1 \qquad \frac{1}{S}\frac{\partial S}{\partial \nu} = \gamma_1$$

give on integration that log S must be of the form $(\alpha + \beta + \gamma)$.

If we substitute the value $(\alpha + \beta + \gamma)$ in (39)

$$\frac{\partial \beta}{\partial \mu}\frac{\partial \gamma}{\partial \nu} + \frac{1}{\nu - \mu}\left(\frac{\partial \beta}{\partial \mu} - \frac{\partial \gamma}{\partial \nu}\right) = 0 \qquad . \quad . \quad . \quad (43)$$

from which either $$\frac{\partial \beta}{\partial \mu} = 0, \quad \frac{\partial \gamma}{\partial \nu} = 0$$

or $$\left(\frac{\partial \beta}{\partial \mu} - \frac{\partial \gamma}{\partial \nu}\right) \Big/ \frac{\partial \beta}{\partial \mu} \cdot \frac{\partial \gamma}{\partial \nu} = \mu - \nu ;$$

the latter alternative gives $$\frac{1}{\frac{\partial \gamma}{\partial \nu}} - \frac{1}{\frac{\partial \beta}{\partial \mu}} = \mu - \nu,$$

hence $$\frac{\partial \beta}{\partial \mu} = -\frac{1}{\mu + A} \qquad \frac{\partial \gamma}{\partial \nu} = -\frac{1}{\nu + A}.$$

Taking account of the symmetrical results for a, we see that the two solutions are *

$$\left.\begin{aligned} S &= 1 \\ \text{or} \quad S &= (\lambda\mu\nu)^{-1} \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad (44)$$

The second solution can be included in the first by the following transformation :—

Let $\lambda_1 = \lambda^{-1}$, $\mu_1 = \mu^{-1}$, $\nu_1 = \nu^{-1}$; then

$$P_1^2 = \left(\frac{\partial x}{\partial \lambda_1}\right)^2 + \left(\frac{\partial y}{\partial \lambda_1}\right)^2 + \left(\frac{\partial z}{\partial \lambda_1}\right)^2 = P^2\left(\frac{\partial \lambda}{\partial \lambda_1}\right)^2 = P^2\lambda^4,$$

and the first three equations of (35) with $S = (\lambda\mu\nu)^{-1}$ become

$$\frac{P_1^2 a_1}{(\lambda_1 - \mu_1)(\lambda_1 - \nu_1)} = \frac{Q_1^2 \beta_1}{(\mu_1 - \lambda_1)(\mu_1 - \nu_1)} = \frac{R_1^2 \gamma_1}{(\nu_1 - \lambda_1)(\nu_1 - \mu_1)} = 1,$$

a_1, β_1, γ_1 being functions of λ_1, μ_1, ν_1 respectively.

* Strictly the second solution is $S^{-1} = (\lambda + A)(\mu + A)(\nu + A)$, but the constant can be absorbed by changing the zero of measurement of λ, μ, ν.

Since this is of the same form as (35)* with S = 1, we see that the two solutions will lead to the same frame of reference, the difference being that in one case certain quantities are taken as the parameters of the surfaces and in the other case their reciprocals.

We can therefore without loss of generality put S = 1.

The second Lamé relation (37) may be written

$$-\frac{1}{P^2}\frac{\partial \log P}{\partial \lambda}\frac{\partial \log Q}{\partial \lambda}-\frac{1}{Q^2}\frac{\partial \log P}{\partial \mu}\frac{\partial \log Q}{\partial \mu}+\frac{1}{R^2}\frac{\partial \log P}{\partial \nu}\frac{\partial \log Q}{\partial \nu}$$
$$+\frac{1}{P^2}\left(\frac{1}{Q}\frac{\partial^2 Q}{\partial \lambda^2}\right)+\frac{1}{Q^2}\left(\frac{1}{P}\frac{\partial^2 P}{\partial \mu^2}\right)=0.$$

Hence by (35) or (38), with S = 1, we have

$$-\frac{1}{P^2}\left\{\left(\frac{1}{2(\lambda-\mu)}+\frac{1}{2(\lambda-\nu)}-\frac{1}{2\alpha_0}\frac{\partial \alpha_0}{\partial \lambda}\right)\frac{1}{2(\lambda-\mu)}+\frac{1}{4(\mu-\lambda)^2}\right\}$$
$$-\frac{1}{Q^2}\left\{\left(\frac{1}{2(\mu-\lambda)}+\frac{1}{2(\mu-\nu)}-\frac{1}{2\beta_0}\frac{\partial \beta_0}{\partial u}\right)\frac{1}{2(\mu-\lambda)}+\frac{1}{4(\mu-\lambda)^2}\right\}$$
$$+\frac{1}{R^2}\frac{1}{4(\nu-\lambda)(\nu-\mu)}=0.$$

Multiplying through by $(\lambda-\mu)^2(\mu-\nu)^2(\nu-\mu)^2$ and substituting for P^2, Q^2, R^2, this becomes

$$(\lambda-\mu)^2\gamma_0=\frac{2}{\lambda-\mu}\{(\nu-\mu)^2(\lambda-\nu)\alpha_0+(\nu-\lambda)^2(\nu-\mu)\beta_0\}$$
$$+(\mu-\nu)^2\left(\alpha_0+(\nu-\lambda)\frac{\partial \alpha_0}{\partial \lambda}\right)+(\lambda-\nu)^2\left(\beta_0+(\nu-\mu)\frac{\partial \beta_0}{\partial \mu}\right) \quad (45)$$

In this equation for γ_0, the right-hand side can be arranged as a polynomial in ν of the third degree. The coefficients must, of course, reduce to constants multiplied by $(\lambda-\mu)^2$, since γ_0 cannot contain λ or μ.

This form of the coefficients requires that the divisor $(\lambda-\mu)$ of the first term should cancel with a similar factor in the numerator. Thus $((\mu-\nu)\alpha_0+(\nu-\lambda)\beta_0)$ must be divisible by $(\lambda-\mu)$. Putting $\lambda=\mu$, we find α_0 must equal β_0, which means that α_0 is the same function of λ that β_0 is of μ. Similarly γ_0 is the same function of ν.

We have therefore shown that α_0, β_0, γ_0 are polynomials of the third degree in λ, μ, ν respectively, the coefficients being the same in each case. It is not difficult to verify that this result satisfies equation (45); but it is unnecessary to do so, because, as we shall see, the resulting expressions for P, Q, R are of a familiar form which is known to satisfy the Lamé relations.

Write accordingly

$$\alpha_0=(\lambda+\epsilon_1)(\lambda+\epsilon_2)(\lambda+\epsilon_3)$$
$$\beta_0=(\mu+\epsilon_1)(\mu+\epsilon_2)(\mu+\epsilon_3)$$
$$\gamma_0=(\nu+\epsilon_1)(\nu+\epsilon_2)(\nu+\epsilon_3)$$

where ϵ_1, ϵ_2, ϵ_3 are constants.

* The last three equations are altered, but they are not required any more.

We obtain finally from (35), since $S = 1$,

$$P^2 = \frac{(\lambda-\mu)(\lambda-\nu)}{(\lambda+\epsilon_1)(\lambda+\epsilon_2)(\lambda+\epsilon_3)} \qquad Q^2 = \frac{(\mu-\lambda)(\mu-\nu)}{(\mu+\epsilon_1)(\mu+\epsilon_2)(\mu+\epsilon_3)}$$

$$R^2 = \frac{(\nu-\lambda)(\nu-\mu)}{(\nu+\epsilon_1)(\nu+\epsilon_2)(\nu+\epsilon_3)} \quad . \quad . \quad . \quad (46)$$

These are the well-known expressions corresponding to confocal quadrics, the quantities $\lambda+\epsilon_1$, etc., being the squares of the axes of the confocals through the point. This establishes the theorem.

It will be noticed that the result is derived from equations which are true whether the system is in a steady state or not, and whether it is moving under its own attraction or under any other field of force.

Summary and Conclusions.

This paper is an investigation of the possible forms of stellar systems in which Schwarzschild's ellipsoidal law of velocities is rigorously obeyed. In general the system is assumed to be in a steady state; but the chief results as to the distribution of velocities hold good also when the system is changing, provided it continues to satisfy the ellipsoidal law.

A possible superposed rotation of the system is considered.

Starting with the consideration of systems of all possible degrees of complexity, it is shown that the ellipsoidal law can be satisfied by only one type, viz., that in which the axes of the velocity-ellipsoids are oriented along the normals to a system of confocal quadrics. The field of discussion is thus narrowed.

The direction of preferential motion (star-streaming) is more or less radial, and the average motion in this direction does not diminish as we pass to the outer parts of the system. The cross-motions decrease as we pass outwards, so that the velocity-ellipsoids become more elongated. (For a more precise statement, see § 5.) These are features of the distribution which were noticed in discussing the particular case of spherical symmetry.

(*a*) *Systems moving solely under their own Attraction.*—The only exact solution is the case of globular symmetry discussed in the second paper (apart from a two-dimensional solution of no interest). No further solution is obtained by admitting a rotation of the system.

(*b*) *Systems moving under Forces other than their own Attraction.*—We may suppose the oblate system of the earlier-type stars to be controlled by the more globular system of late-type stars; or that the lucid stars move mainly under the gravitational field of a cosmic gas (producing absorption of light in space), their own attractions being insignificant in comparison. If, in either case, we take the controlling system to be globular, we evade the chief difficulty as to how its own equilibrium is maintained. Under a

controlling distribution, we can obtain an oblate distribution of the stars, either by taking prolate principal velocity-surfaces, or by superposing a rotation of the whole system, or by a combination of the two causes. It appears to be possible to construct a system in this way in fairly good quantitative accordance with the chief results of observation, though one or two points remain doubtful.

It is not easy to estimate how far the results would be modified by admitting small deviations from the ellipsoidal law (such as are indicated by the observations). In the case of a system moving under its own attraction, where there is no exact solution, it is possible that an approximate solution might suffice, if we relaxed this condition slightly. It is certainly not permissible to assert that the properties of these mathematical stellar systems must necessarily be found in the actual universe; but, if we do not misuse it, an ideal system can help towards a better understanding of the interrelations of the phenomena.

Erratum in the "Second Paper."

Mr. Hartley points out to me that at the beginning of § 5 (vol. lxxv. p. 370) I have integrated with respect to u from $-\infty$ to ∞, whereas I have previously treated u as a signless quantity (introducing a factor 2 on p. 368 to allow for this). The integration should have been from 0 to ∞, and the necessary correction can be made by writing $\frac{B}{2}$ for B in equations (14) and (15). The factor 2 is then absorbed in the choice of units along with B, and the rest of the work stands without alteration.

Note on Professor Dale's Method of Periodic Analysis.

By H. C. Plummer, M.A.

The method proposed by Professor Dale (*M.N.*, lxxiv. p. 628) for the resolution of a compound periodic function is of great interest. It may be pointed out that the main propositions on which the method rests are capable of a simpler and more direct proof.

1. In place of the operator E, which has another meaning in finite differences, we may use M to denote the mean of the preceding and following tabular entry, so that

$$Mu_n = \tfrac{1}{2}(u_{n-1} + u_{n+1}).$$

Since secular terms can be removed by preliminary differencing, the function may be taken to be

$$f(\theta) = \Sigma A \sin(p\theta + \alpha)$$

with r distinct components. Let t be the common difference of the tabular argument. Then

$$u_n = \Sigma A \sin(npt + \alpha)$$
$$Mu_n = \Sigma A \sin(npt + \alpha) \cos pt$$
$$\ldots\ldots$$
$$M^r u_n = \Sigma A \sin(npt + \alpha) \cos^r pt.$$

The result of eliminating $A \sin(npt + \alpha)$, . . . may be written

$$\begin{vmatrix} M_n^0 & 1 & \ldots & 1 \\ M_n^1 & \cos p_1 t & \ldots & \cos p_r t \\ \cdot & \cdot & \cdots & \cdot \\ M_n^r & \cos^r p_1 t & \ldots & \cos^r p_r t \end{vmatrix} = 0.$$

This equation holds for all values of n, and also for the means of numbers constructed with any range of values of n, since the equation is linear in M_n. It may be written

$$M_n^0 N_0 + M_n^1 N_1 + \ldots + M_n^r N_r = 0$$

and r different values of n (1, 2 . . .) give

$$M_1^0 N_0 + M_1^1 N_1 + \ldots + M_1^r N_r = 0$$
$$\ldots$$
$$M_r^0 N_0 + M_r^1 N_1 + \ldots + M_r^r N_r = 0.$$

But if $x = \cos pt$, where p has any of the values $p_1, \ldots, p_r$,

$$N_0 + xN_1 + \ldots + x^r N_r \equiv 0.$$

Thus we have $(r+1)$ equations from which $N_0, N_1, \ldots, N_r$ can be eliminated. The result is

$$\begin{vmatrix} M_1^0 & \ldots & M_r^0 & 1 \\ M_1^1 & \ldots & M_r^1 & x \\ \ldots & \ldots & \cdot & \cdot \\ M_1^r & \ldots & M_r^r & x^r \end{vmatrix} = 0.$$

and the r roots give the r values of $\cos pt$. This is Professor Dale's equation for the periods.

2. Again, the equations for the coefficients now become

$$M_n^0 = \Sigma A \sin(npt + \alpha)$$
$$M_n^1 = \Sigma A \sin(npt + \alpha) x$$
$$\ldots$$
$$M_n^{r-1} = \Sigma A \sin(npt + \alpha) x^{r-1}.$$

Hence

$$A_s \sin(np_s t + \alpha_s) \begin{vmatrix} 1 & \ldots & 1 & \ldots & 1 \\ x_1 & \ldots & x_s & \ldots & x_r \\ \cdot & \ldots & \cdot & \ldots & \cdot \\ x_1^{r-1} & \ldots & x_s^{r-1} & \ldots & x_r^{r-1} \end{vmatrix} = \begin{vmatrix} 1 & \ldots & M_n^0 & \ldots & 1 \\ x_1 & \ldots & M_n^1 & \ldots & x_r \\ \cdot & \ldots & \cdot & \ldots & \cdot \\ x_1^{r-1} & \ldots & M_n^{r-1} & \ldots & x_r^{r-1} \end{vmatrix}$$

If the determinant on the right be written

$$\Delta_s = X_s(M_n^{r-1} + s_1 M_n^{r-2} + \ . \ . \ . \ + s_{r-1} M_n^{0})$$

in which x_s does *not* occur, the determinant on the left is

$$\begin{aligned}\Delta &= X_s(x_s^{r-1} + s_1 x_s^{r-2} + \ . \ . \ . \ + s_{r-1}) \\ &= X_s(x_s - x_1) \ . \ . \ . \ (x_s - x_r)\end{aligned}$$

since every difference between two values of x is a factor of Δ. Hence $s_1, s_2, \ . \ . \ . \ s_{r-1}$ are the coefficients of powers of x in the product $(x-x_1) \ . \ . \ . \ (x-x_r)$, from which the factor $(x-x_s)$ is absent. Hence they can be found simply by dividing the period equation by $(x-x_s)$. We now have

$$\begin{aligned}A_s \sin(np_s t + \alpha_s) &= \frac{M_n^{r-1} + s_1 M_n^{r-2} + \ . \ . \ . \ + s_{r-1} M_n^{0}}{(x_s - x_1) \ . \ . \ . \ (x_s - x_r)} \\ &= P_s(n).\end{aligned}$$

Similarly, with another value of n, say m, we have

$$A_s \sin(mp_s t + \alpha_s) = P_s(m)$$

and it follows that

$$\begin{aligned}A_s \cos\alpha_s \ . \ \sin(n-m)p_s t &= P_s(n)\cos mp_s t - P_s(m)\cos np_s t \\ A_s \sin\alpha_s \ . \ \sin(n-m)p_s t &= P_s(m)\sin np_s t - P_s(n)\sin mp_s t.\end{aligned}$$

3. The latter results are not directly comparable with Professor Dale's, because his process includes the removal of a constant term by one preliminary differencing. This step is not absolutely necessary, for the constant term should correspond to the root 1 in the period equation. In order to make the results comparable, let the original function be

$$v_n = c + \Sigma a_s \sin(np_s t + \beta_s).$$

Then

$$u_n = v_{n+1} - v_n = \Sigma a_s \cos\{(n+\tfrac{1}{2})p_s t + \beta_s\} \ . \ 2\sin\tfrac{1}{2}p_s t$$

and hence

$$\begin{aligned}A_s \sin\alpha_s &= \ \ a_s \cos(\tfrac{1}{2}p_s t + \beta_s) \ . \ 2\sin\tfrac{1}{2}p_s t \\ A_s \cos\alpha_s &= -a_s \sin(\tfrac{1}{2}p_s t + \beta_s) \ . \ 2\sin\tfrac{1}{2}p_s t\end{aligned}$$

or

$$\begin{aligned}a_s \cos\beta_s \ . \ 2\sin\tfrac{1}{2}p_s t &= \ \ A_s \sin(\alpha_s - \tfrac{1}{2}p_s t) \\ a_s \sin\beta_s \ . \ 2\sin\tfrac{1}{2}p_s t &= -A_s \cos(\alpha_s - \tfrac{1}{2}p_s t).\end{aligned}$$

It now follows that

$$\begin{aligned}a_s \cos\beta_s &= \frac{P_s(m)\sin(n+\frac{1}{2})p_s t - P_s(n)\sin(m+\frac{1}{2})p_s t}{2\sin\frac{1}{2}p_s t \sin(n-m)p_s t} \\ a_s \sin\beta_s &= \frac{P_s(m)\cos(n+\frac{1}{2})p_s t - P_s(n)\cos(m+\frac{1}{2})p_s t}{2\sin\frac{1}{2}p_s t \sin(n-m)p_s t}.\end{aligned}$$

But for the difference of notation, these agree with Professor Dale's results, when we substitute $n-1$ for m.

4. The theorem given by Professor Dale in an appendix is not required in the above form of proof. But it can be deduced with great facility. Any set of $(2r+1)$ constants, $a_{n-r}, \ldots a_n, \ldots a_{n+r}$, can be expressed by the form

$$a_n = \Sigma A\lambda^n ,$$

where λ has $(2r+1)$ distinct values, since there are $(2r+1)$ equations to determine the $(2r+1)$ quantities A. Then since

$$a_{n+r} + a_{n-r} = \Sigma A\lambda^n(\lambda^r + \lambda^{-r}),\ 2Ma_n = \Sigma A\lambda^n(\lambda + \lambda^{-1})$$

$$\begin{aligned} &p_0(a_{n+r} + a_{n-r}) + p_1(a_{n+r-1} + a_{n-r+1}) + \ldots + p_r a_n \\ &\quad = \Sigma A\lambda^n\{p_0(\lambda_r + \lambda^{-r}) + p_1(\lambda^{r-1} + \lambda^{-r+1}) + \ldots + p_r\} \\ &\quad = \Sigma A\lambda^n\{q_0(\lambda + \lambda^{-1})^r + q_1(\lambda + \lambda^{-1})^{r-1} + \ldots + q_r\} \\ &\quad = 2^r q_0 M^r a_n + 2^{r-1} q_1 M^{r-1} a_n + \ldots + q_r a_n. \end{aligned}$$

This is the equivalent, in slightly different notation, of Professor Dale's theorem. The relation between the set q and the set p is expressed otherwise by putting

$$\cos\theta = \tfrac{1}{2}(\lambda + \lambda^{-1}), \quad \cos r\theta = \tfrac{1}{2}(\lambda^r + \lambda^{-r})$$

and the transformation is then defined by the identity

$$\begin{aligned} &2p_0 \cos r\theta + 2p_1 \cos(r-1)\theta + \ldots + p_r \\ &= q_0(2\cos\theta)^r + q_1(2\cos\theta)^{r-1} + \ldots + q_r. \end{aligned}$$

The operator M is preferred to Professor Dale's E $(=2M)$ because it does not introduce quantities of an order greater than those in the original table. The mere repeated application of this operator to a given series may render good service owing to its fundamental property of reducing terms with different periods in different proportions, so that ultimately the function to be studied may be simplified by the practical elimination of certain components.

Magnetic Disturbances (1904–1913) *as recorded at the Royal Observatory, Greenwich, and their Association with the Solar Rotation.* Fourth Paper. By E. Walter Maunder.

The present paper is in continuation of a series of three papers under a similar title that were communicated to the Royal Astronomical Society in 1904 and 1905, and were published in vol. lxv. of the *Monthly Notices* at pp. 2, 538, and 666 respectively. The tables contained in the present paper, therefore, have been numbered forward from the last table in the earlier series.

Table I. (vol. lxv., p. 4) contained a catalogue of the magnetic disturbances of 20′ and upwards in declination, as given in the

plates of the *Greenwich Observations* from 1882 to 1903; Table XX. in the present paper continues that catalogue from 1904 to 1913. The earlier catalogue, as mentioned in the first paper, was practically one of the magnetic disturbances for which the photographic traces had been reproduced in the plates published in the *Greenwich Observations*, and these plates represented very nearly all the disturbances observed at Greenwich in which the greatest recorded magnetic movement exceeded 20′ in declination or 100 in horizontal force (c.g.s. value × 10^5). Similarly, the catalogue in Table XX. is based primarily upon the plates published in the *Greenwich Observations*, but, as in late years, fewer of the small disturbances have been thus represented, those of simpler character being merely described in the notes, the original photographic sheets have been carefully examined, and several disturbances above the 20′ amplitude of movement, but not shown in the Greenwich plates, have also been included in the catalogue. These are indicated by an asterisk. In five instances disturbances have been represented in the Greenwich plates although the greatest recorded magnetic movement fell short of the 20′ amplitude that was generally adopted as the inferior limit.

The first column in Table XX. gives the reference number of the disturbance, and this is in continuance of the similar number in Table I. The second column gives the rank of the disturbance: the letters G, V, A, and M signifying "Great," "Very Active," "Active," and "Moderate" respectively, and corresponding to magnetic movements exceeding 60′, 40′, 30′, or 20′ in amplitude respectively. The third column gives the date and hour of the commencement of the disturbance; and the fourth its duration in hours. Column five, headed "Beginning," contains the letter S in those instances where the movement began with a characteristic sharp to-and-fro twitch. In these cases the time of beginning is given to the tenth of an hour. Column six records the extreme amplitude of movement in declination. The two last columns give the number of the solar rotation on Carrington's numeration (adopted in the *Greenwich Photoheliographic Results*) and the heliographic longitude of the centre of the Sun's disc at the time of the commencement of the magnetic disturbance.

An examination of this catalogue, as of those which preceded it, shows that there is a strong tendency for disturbances to recur when the same meridian has returned to the centre of the Sun's disc, that is to say, after the interval of a synodic rotation of the Sun. Out of 90 disturbances spread over 134 rotation periods, 50 fall into "sequences" of this character, of which 2 consist of 5 members each, and 3 of 3 members.

These "sequences" illustrate what I called in the earlier papers the "Interval-Relation"; in other words, there is a marked tendency for magnetic disturbances to recur after an *interval* equal to the synodic rotation of the Sun period, as determined from the observation of sun-spots. But the "Distribution-Relation," or tendency of magnetic disturbances to occur when certain *particular*

meridians are on the centre of the Sun's disc, is also apparent from the present catalogue. Thus 17 out of the 90 disturbances commenced when longitudes 343° to 359° were on the centre of the disc; and the great majority of the sequences, 16 out of 19, correspond to the hemisphere 194° to 14°. The present catalogue, therefore, offers the same features as did the catalogues given in Table I. (for the years 1882 to 1903) and in Table IX. (for the years 1848 to 1881) of the earlier papers.

8 *Maze Hill, Greenwich Park, S.E.:*
1915 *October* 20.

TABLE XX.

Magnetic Disturbances of 20′ *and upwards in Declination, recorded in the "Greenwich Magnetical and Meteorological Observations,"* 1904 *to* 1913.

Reference No.	Class.	Greenwich Civil Time of Commencement of Disturbance.			Duration in Hours.	Beginning.	Extreme Amplitude of Movement in Declination.	No. of Rotation.	Heliogr. Long. of Centre of Sun's Disc.
		1904.	d	h	h		′		°
277	M	Jan.	28	16	7	...	26	673	206·1*d*
278	M	April	1	1	24	...	26	675	91·1
279	M		17	16·4	36	S	26	676	231·3
280	A	June	15	16·5	21	S	31	678	171·1
281	M	July	6	20·9	9	S	20	679	250·7
282	M	Aug.	3	13·9	21	S	26	680	244·1
283	M	Sept.	24	19·5	32	...	25	682	274·1
284	A	Oct.	21	17	9	...	31	683	279·3
		1905.							
285	A	Jan.	5	15	10	...	31	686	358·8
286	V	Feb.	3	1·7	26	S	46	687	344·2
287	M	Mar.	2	13·3	13	S	28	688	342·3
288	M		7	3·0	12	S	20	...	282·1
289	A	April	1	1·2	28	S	39	689	313·4
290	M	July	5	22	20	...	25	692	125·8
291	M	Aug.	2	2	19	...	20	693	126·3
292	A	Nov.	12	8·1	18	S	32	697	216·5
293	V		15	15·3	32	S	58	...	173·1
		1906.							
294	M	Jan.	31	16	8	...	22	700	238·3
295	V	Feb.	24	17	42	...	44	701	281·7
296	A	July	11	13·4	19	S	30	706	273·6
297	M	Aug.	7	19·5	27	S	21	707	273·0
298	A	Sept.	22	18	10	...	32	708	26·2
299	M	Nov.	21	18	15	...	20	711	314·8
300	V	Dec.	21	22	33	...	45	712	277·4

d signifies that the register is defective.

TABLE XX.—*continued.*

Reference No.	Class.	Greenwich Civil Time of Commencement of Disturbance.	Duration in Hours.	Beginning.	Extreme Amplitude of Movement in Declination.	No. of Rotation.	Heliogr. Long. of Centre of Sun's Disc.
		1907. d h	h		′		°
301*	M	Jan. 11 8·8	14	S	29	...	8·0
302	A	14 19·6	17	S	35	713	322·6
303	M	Feb. 7 8·1	21	S	28	...	12·9
304	G	9 14·2	15	S	100+	714	343·2
305	M	11 19	9	...	23	...	314·1
306	M	Mar. 10 5·1	29	S	29	715	326·2
307	V	11 17·4	17	S	45	...	306·3
308	A	21 13·4	19	S	33	...	176·7
309	M	May 18 14·0	17	S	25	717	130·3
310	M	28 20	22	...	25	718	354·7
311	M	June 19 15	19	...	24	...	66·3
312	M	July 10 14·4	21	S	26	719	148·7
313	M	27 18	17	...	23	720	281·8
314	A	Sept. 10 1·8	34	S	37	721	56·0
315	M	Oct. 13 7·8	65	S	29	723	337·1
316	A	Nov. 21 10·7	9	S	32	724	181·3
		1908.					
317	V	Mar. 26 17	44	...	46	729	318·0
318	M	May 25 18	16	...	27	731	244·7
319*	M	July 15 15	21	...	25	733	291·4
320	M	Aug. 8 20	24	...	29	734	331·2
321*	M	11 17	33	...	24	...	293·2
322	M	21 8·6	22	S	25	...	165·6
323*	M	Sept. 5 0	28	...	30±	735	332·2*d*
324	G	11 21·8	22	S	80+	...	241·0
325	G	29 1·5	60	S	60+	...	14·6*d*
326*	A	Oct. 12 17	29	...	33	736	194·5
327*	A	Nov. 8 0	27	...	32	737	207·8
328*	A	Dec. 4 16	20	...	31	738	216·3
		1909.					
329*	M	Jan. 1 14	7	...	29	739	208·5
330	G	3 12·9	39	S	70	...	182·8
331	V	29 12	58	...	45	740	200·9
332	G	May 14 4·9	39	S	50	744	259·9
333	A	18 5·1	26	S	30	...	206·9
334*	M	Aug. 28 17	81	...	20	748	291·0

d signifies that the register is defective.

TABLE XX.—*continued*.

Reference No.	Class.	Greenwich Civil Time of Commencement of Disturbance.			Duration in Hours.	Beginning.	Extreme Amplitude of Movement in Declination.	No. of Rotation.	Heliogr. Long. of Centre of Sun's Disc.
		1909.	d	h	h		′		°
335*	M	Sept.	21	12	16	...	27	749	336·8
336	G		25	8·5	18	S	200±	...	285·9
337	A		30	4·0	18	S	30	...	222·4
338	V	Oct.	18	16	33	...	42	750	338·4
339*	M		23	0·0	21	S	25	...	281·2
		1910.							
340	V	Mar.	27	18	35	...	45	755	29·1
341*	A		30	21	75	...	31	756	347·9
342*	M	April	23	9	17	...	25	...	37·7
343*	M		26	16·6	10	S	22	757	353·9
344	V	June	20	1	21	...	45	759	354·9
345	M	Aug.	21	14	32	...	25	761	247·5
346	V	Sept.	29	8·0	16	S	46	762	95·9
347	A	Dec.	28	9	49	...	33	766	348·8
		1911.							
348	A	Jan.	24	10	19	...	36	767	352·7*d*
349*	A	Feb.	13	19	11	...	30	...	84·5
350	A		20	21	51	...	33	768	351·2
351*	A	Mar.	20	3	23	...	32	769	352·1
352	A	April	8	20	19	...	37	...	92·2
353	M		16	8·1	27	S	26	770	353·1
354	M	May	14	20	39	...	18	771	336·5
355	V	Oct.	10	17	18	...	46	776	168·2
356	M	Dec.	11	1	25	...	25	778	79·6
		1912.							
357	M	Mar.	8	17	11	...	15	782	351·9
358	M	April	5	11	18	...	26	783	346·0
359	M	Aug.	5	21	23	...	20	787	166·9
360	V	Sept.	17	14·0	15	S	43	789	322·6
361	A	Oct.	14	16	10	...	32	790	325·3
362	M	Dec.	6	22	26	..	17	792	343·2
		1913.							
363	M	Jan.	3	4	17	...	15	793	344·3
364	M	Mar.	14	4·5	20	S	22	795	142·1
365	M	April	9	12	15	...	29	796	155·1
366	M	June	1	6	24	...	15	798	177·7

d signifies that the register is defective.

TABLE XXI.

Sequences of Magnetic Disturbances in Consecutive Rotations.

Reference No. of Sequence.	No. of Rotation.	Reference No. of Disturbance.	Heliogr. Long. of Centre of Sun's Disc.
			°
LXX	679	281	250·7
	680	282	244·1
LXXI	682	283	274·1
	683	284	279·3
LXXII	686	285	358·8
	687	286	344·2
	688	287	342·3
LXXIII	692	290	125·8
	693	291	126·3
LXXIV	706	296	273·6
	707	297	273·0
LXXV	711	299	314·8
	712	...	...
	713	302	322·6
	714	305	314·1
	715	306	326·2
LXXVI	712	301	8·0
	713	303	12·9
LXXVII	733	319	291·4
	734	321	293·2
LXXVIII	734	320	331·2
	735	323	332·2
LXXIX	736	326	194·5
	737	327	207·8
	738	328	216·3
	739	329	208·5
	740	331	200·9
LXXX	748	334	291·0
	749	336	285·9
	750	339	281·2
LXXXI	749	335	336·8
	750	338	338·4
LXXXII	755	340	29·1
	756	342	37·7
LXXXIII	756	341	347·9
	757	343	353·9
	758	...	...
	759	344	354·9
LXXXIV	766	347	348·8
	767	348	352·7
	768	350	351·2
	769	351	352·1
	770	353	353·1
LXXXV	782	357	351·9
	783	358	346·0
LXXXVI	789	360	322·6
	790	361	325·3
LXXXVII	792	362	343·2
	793	363	344·3
LXXXVIII	795	364	142·1
	796	365	155·1
	797	...	...
	798	366	177·7

Erratum in Annual Report, 1915, vol. lxxv.

Page 329, line 6 from bottom:

for O. W. Walkey *read* O. R. Walkey.